



Algorithms and Data Structures

Winter Term 2020/2021

Exercise Sheet 1

Exercise 1: Bubblesort

The following pseudocode describes the BUBBLESORT algorithm with input array A of length n .

Algorithm 1 BUBBLESORT($A[0 \dots n-1]$)

```
for  $i = 0$  to  $n - 2$  do
  for  $j = 0$  to  $n - 2$  do
    if  $A[j] > A[j+1]$  then
      SWAP( $j, j+1$ )
```

▷ operation SWAP($j, j+1$) swaps array entries $A[j]$ and $A[j+1]$

- (a) Assume BUBBLESORT runs on input $A = [24, 9, 15, 11, 4, 21]$. Give A after the end of each iteration of the outer for-loop.
- (b) Argue why BUBBLESORT is correct (i.e., array A is always sorted after the algorithm is finished).

Exercise 2: Counting Sort

The following pseudocode describes the COUNTINGSORT algorithm which receives an array $A[0 \dots n-1]$ as input containing values in $[0..k]$. Additionally there is an Array $\text{counts}[0 \dots k]$ initialized with 0.

Algorithm 2 COUNTINGSORT(A, COUNTS) ▷ integer arrays $A[0 \dots n-1], \text{counts}[0 \dots k]$

```
for  $i \leftarrow 0$  to  $n - 1$  do
  counts[ $A[i]$ ] ++
```

▷ ++ is the increment operation

```
 $i \leftarrow 0$ 
for  $j \leftarrow 0$  to  $k$  do
  for  $\ell \leftarrow 1$  to counts[ $j$ ] do
     $A[i] \leftarrow j$ 
     $i ++$ 
```

- (a) Assume COUNTINGSORT runs on input $A = [5, 2, 3, 0, 5, 3, 4, 2, 5, 0, 1, 3, 5, 0, 0]$. Give A and counts after the algorithm has terminated.
- (b) Argue why COUNTINGSORT is correct (i.e., the algorithm has sorted array A after finishing).

Exercise 3: Implementation

- (a) Implement one of the above two algorithms in a programming language of your choice (in the lecture and exercise class we will see/use Python).¹
- (b) Test your implementation with random inputs as follows. Generate input arrays *of length 10, 30, 100, 200, 300, 500, 700, and 1000* respectively, each *filled with randomly generated integer values ranging from 0 to 200*. Run the algorithm on each input and check the correctness.
- (c) Implement some functionality to measure the elapsed time of the algorithm from start to finish (e.g., by using the python-module *time*). Run the algorithm again with the above inputs and note down the elapsed times. What do you think is the dependency of the running time on n (and k , in case of the COUNTINGSORT algorithm)?

¹As a side-note: In this course we assume that you have some (very) basic programming skills, enabling you to implement short pseudo codes like the ones given above in a programming language of your choice. Since this course is more on the theoretical side, we will not ask much more than that in terms of programming skills. If you never attended some programming-course and/or experience difficulties to implement the above algorithms, please try to catch up using literature, tutorials and/or contact us.