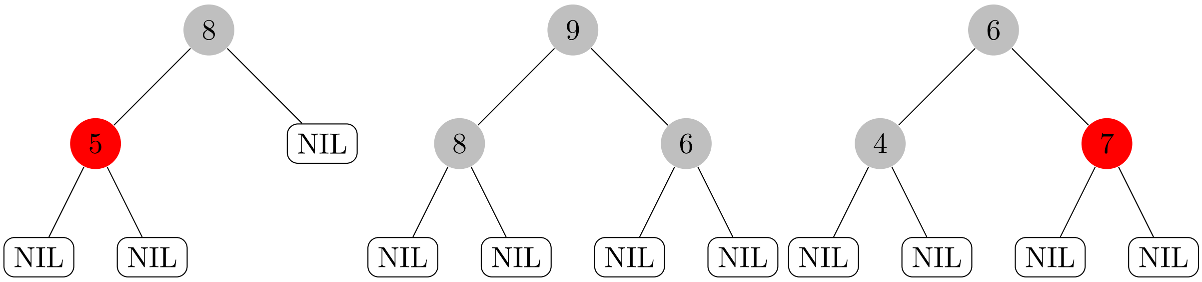
University of Freiburg
Dept. of Computer Science
Prof. Dr. F. Kuhn

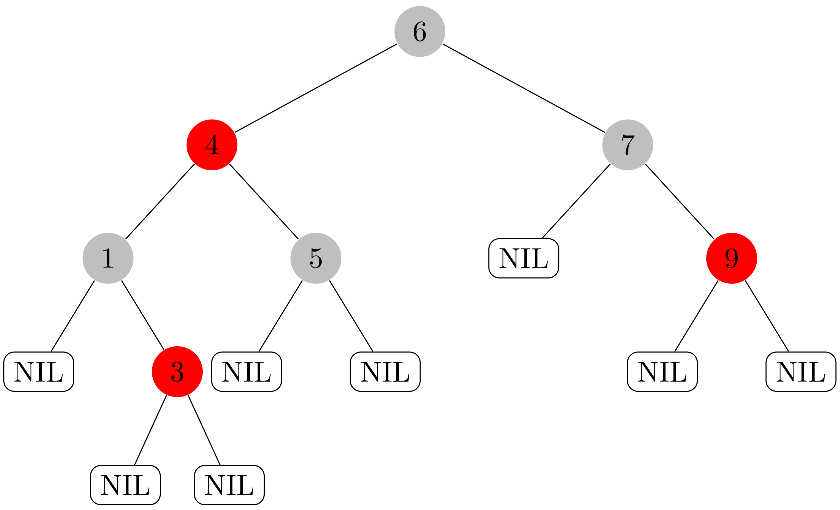
Algorithms and Data Structures
Winter Term 2020/2021
Exercise Sheet 7

Exercise 1: Red-Black Trees

(a) Decide for each of the following trees if it is a red-black tree and if not, which property is violated:



(b) On the following red-black tree, first execute the operation `insert`(8) and afterwards `delete`(5). Draw the resulting tree and document intermediate steps.



Exercise 2: AVL-Trees

An AVL-tree is a binary search tree with the additional property that for each node v, the depth of its left and its right subtree differ by at most 1.

(a) Show via induction that an AVL-tree of height d is filled completely up to depth $\lfloor \frac{d}{2} \rfloor$.

A binary tree is filled completely up to depth d' if it contains for all $x \leq d'$ exactly 2^x nodes of depth x.

(b) Give a recursion relation that describes the minimum number of nodes of an AVL-tree as a function of d.

(c) Show that an AVL-tree with n nodes has depth $\mathcal{O}(\log n)$.

You can either use part (a) or part (b).