University of Freiburg
Dept. of Computer Science
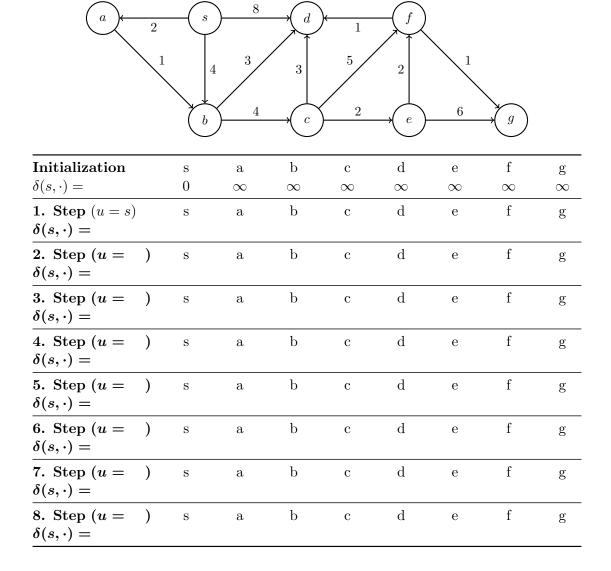Prof. Dr. F. Kuhn

Algorithms and Data Structures
Winter Term 2020/2021
Exercise Sheet 10

Exercise 1: Dijkstras' Algorithm

Execute Dijkstras' Algorithm on the following weighted, directed graph, starting at node s. Into the table further below, write the distances from each node to s that the algorithm stores in the priority queue after each iteration.

	s	a	b	c	d	e	f	g
Initialization $\delta(s,\cdot) =$	0	∞	∞	∞	∞	∞	∞	∞
1. Step $(u = s)$ $\boldsymbol{\delta(s,\cdot) =}$	s	a	b	c	d	e	f	g
2. Step $(u = \quad)$ $\boldsymbol{\delta(s,\cdot) =}$	s	a	b	c	d	e	f	g
3. Step $(u = \quad)$ $\boldsymbol{\delta(s,\cdot) =}$	s	a	b	c	d	e	f	g
4. Step $(u = \quad)$ $\boldsymbol{\delta(s,\cdot) =}$	s	a	b	c	d	e	f	g
5. Step $(u = \quad)$ $\boldsymbol{\delta(s,\cdot) =}$	s	a	b	c	d	e	f	g
6. Step $(u = \quad)$ $\boldsymbol{\delta(s,\cdot) =}$	s	a	b	c	d	e	f	g
7. Step $(u = \quad)$ $\boldsymbol{\delta(s,\cdot) =}$	s	a	b	c	d	e	f	g
8. Step $(u = \quad)$ $\boldsymbol{\delta(s,\cdot) =}$	s	a	b	c	d	e	f	g

Exercise 2: Currency Exchange

Consider n currencies $w_1, \ldots, w_n$. The exchange rates are given in an $n \times n$-matrix A with entries a_{ij} $(i, j \in \{1, \ldots, n\})$. Entry a_{ij} is the exchange rate from w_i to w_j, i.e., for one unit of w_i one gets a_{ij} units of w_j.

Given a currency w_{i_0}, we want to find out whether there is a sequence $i_0, i_1, \ldots, i_k$ such that we make profit if we exchange one unit of w_{i_0} to w_{i_1}, then to w_{i_2} etc. until w_{i_k} and then back to w_{i_0}.

(a) Translate this problem to a graph problem. That is, define a graph and a property which the graph fulfills if and only if there is a sequence of currencies as described above.

(b) Give an algorithm that decides in $\mathcal{O}(n^3)$ time steps whether there is a sequence of currencies as described above. Explain the correctness and runtime.

   Hint: $\log(a \cdot b) = \log a + \log b$.