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Algorithms Theory
Exercise Sheet 9

Due: Tuesday, 19th of January 2021, 4 pm

Exercise 1: Matching vs Vertex Cover *(10 Points)*

Given an undirected Graph $G = (V, E)$, a *vertex cover* of G is a set of nodes $S \subseteq V$ such that for all $\{u, v\} \in E$, we have $\{u, v\} \cap S \neq \emptyset$. A minimum vertex cover is a vertex cover of minimum size.

a) Show that for a maximum matching M^* and a minimum vertex cover S^* we have $|M^*| \leq |S^*|$. *(2 Points)*

Next we want to show that in bipartite graphs, it also holds $|S^*| \leq |M^*|$.

b) Recall that we can solve the maximum bipartite matching problem by reduction to maximum flow. Also recall that if we are given a maximum matching M^* (and thus a maximum flow of the corresponding flow network), we can find a minimum s-t cut by considering the residual graph. Describe how such a minimum cut looks like.

   Hint: Consider the set of all nodes which can be reached from an unmatched node on the left side via an alternating path. *(3 Points)*

c) Use the above description to show that any bipartite graph G has a vertex cover S^* of size $|M^*|$. *(3 Points)*

d) Show that the same thing is not true for general graphs by showing that for every $\varepsilon > 0$, there exists a graph $G = (V, E)$ for which $|S^*| \geq (2 - \varepsilon)|M^*|$.

   Hint: First try to find any graph for which $|S^*| > |M^*|$. *(2 Points)*

Exercise 2: Contention Resolution *(10 Points)*

Show that for the randomized algorithm for contention resolution from the lecture, the expected time until all processes have been successful is $O(n \log n)$.