University of Freiburg
Dept. of Computer Science
Prof. Dr. F. Kuhn
P. Bamberger, P. Schneider

Algorithms Theory
Exercise Sheet 11

Due: Tuesday, 2nd of February 2021, 4 pm

Exercise 1: Contraction Algorithm *(10 Points)*

(a) We adjust the contraction algorithm from the lecture in the following way: Instead of contracting a uniform random edge, we choose a uniform random pair of remaining nodes in each step and merge them. That is, as long as there are more than two nodes remaining, we choose two nodes $u \neq v$ uniformly at random and replace them by a new node w. For all edges $\{u, x\}$ and $\{v, x\}$ we add an edge $\{w, x\}$ and remove self-loops created at w.

Is this a reasonable approach? Explain your answer. *(6 Points)*

(b) The edge contraction algorithm has a success probability $\geq 1/\binom{n}{2}$. We used properties of this algorithm to show that there are at most $\binom{n}{2}$ minimum cuts in any graph. The improved (recursive) min-cut algorithm has a success probability $\geq 1/\log n$. Why can't we use the same argumentation to show that there are at most $\log n$ minimum cuts in any graph (which clearly isn't true as we have seen that cycles have $\binom{n}{2}$ minimum cuts). *(4 Points)*

Exercise 2: Edge Connectivity *(10 Points)*

Given a graph $G = (V, E)$ with edge connectivity $\lambda(G)$ and a parameter $\varepsilon \in (0, 1)$, we obtain a graph $H = (V, F)$ by adding each edge from E independently with probability p to F. Show that for every constant $c > 0$ there is a constant $d > 0$ such that for $p \geq \frac{d \ln n}{\varepsilon^2 \lambda(G)}$, we have $\lambda(H) = (1 \pm \varepsilon) \cdot p \cdot \lambda(G)$ with probability at least $1 - \frac{1}{n^c}$.

Hint: Use Chernoff's bound (Chapter 7, Part IV, page 7) and the Cut Counting theorem (Chapter 7, Part VIII, page 7) for general $\alpha \geq 1$.