Albert-Ludwigs-Universität, Inst. für Informatik
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Theoretical Computer Science - Bridging Course
Winter Term 2020/21
Exercise Sheet 1

In case you wish to get feedback, submit electronically by 12:15, Monday, November 9.

Exercise 1: Proof by Induction *(3 Points)*

Prove by induction for any positive integer number $n, n^3 + 2n$ is divisible by 3.

Exercise 2: Which Statement is True? *(2+2+3 Points)*

Let A, B and C be sets in some universal set U. Which of the following statements is always true. Justify.

1. If $A \cap B = A \cap C$, then $B = C$.
2. If $A \cup B = A \cup C$, then $B = C$.
3. $\overline{A \cup B} = \overline{A} \cap \overline{B}$. *Remark:* $\overline{A}$ *is the compliment of* A

Exercise 3: Visiting All Nodes (Part1) *(3 Points)*

A *simple graph* is a graph without self loops, i.e. every edge of the graph is an edge between two distinct nodes. A *complete graph* is a simple undirected graph in which every pair of distinct nodes is connected by a unique edge e.g. a triangle on 3 nodes.

Prove that every complete graph G has a path P that visits all the nodes of G.

Exercise 4: Visiting All Nodes (Part 2) *(7 Points)*

A *directed path* P on n vertices is a simple directed graph whose edge set is the following set of ordered pairs $\{(v_i, v_{i+1}) \mid 1 \leq i \leq n-1 \text{ and } v_i \text{ is a node in } P\}$ i.e. a path in which all the arrows point in the same direction as its steps . We write $P = v_1v_2...v_n$ to denote the directed path P.
A *tournament* is an orientation of a complete graph, or equivalently a directed graph in which every pair of distinct vertices is connected by a directed edge with any one of the two possible orientations.

Prove that every tournament T has a directed path P that visits all the nodes of T.

Hint: Prove by contradiction. Consider a longest directed path in T *and suppose that this path doesn't visit all nodes in* T*. What happens then?*

Bonus Question

The degree $d(v)$ of a node $v \in V$ of an undirected graph $G = (V, E)$ is the number of its neighbors, i.e,

$$d(v) = |\{u \in V \mid \{v, u\} \in E\}|.$$

Show that the number of vertices of odd degree must be even.
Hint: Recall the Handshaking lemma $\sum_{v \in V} d(v) = 2|E|$.