

Theoretical Computer Science - Bridging Course

Winter Term 2020/21

Exercise Sheet 10

for getting feedback submit electronically by 12:15, Monday, January 25, 2021

Exercise 1: Resolution Calculus

(3+3 Points)

Considering each of the following cases, first convert the knowledge base (KB_i) and the formula (φ_i) to CNFs. Then, by resolution, show that the knowledge base entails the formula.

(a) $KB_1 := \{(x \wedge y) \rightarrow (z \vee w), y \rightarrow x, (z \wedge y) \rightarrow 0, y\}$

$$\varphi_1 := w \wedge y$$

(b) $KB_2 := \{\neg A \rightarrow B, B \rightarrow A, A \rightarrow (C \wedge D)\}$

$$\varphi_2 := A \wedge C \wedge D$$

Exercise 2: Implication vs. Entailment

(5 Points)

Show that $P \models Q \leftrightarrow (True \models P \rightarrow Q)$

Exercise 3: Understanding First Order Logic

(2+2+2 Points)

Consider the following **first order logical** formulae

$$\varphi_1 := \forall x R(x, x)$$

$$\varphi_2 := \forall x \forall y R(x, y) \rightarrow (\exists z R(x, z) \wedge R(z, y))$$

$$\varphi_3 := \exists x \exists y (\neg R(x, y) \wedge \neg R(y, x))$$

where x, y are variable symbols and R is a binary predicate. Give an interpretation

(a) I_1 which is a **model** of $\varphi_1 \wedge \varphi_2$.

(b) I_2 which is **no model** of $\varphi_1 \wedge \varphi_2 \wedge \varphi_3$.

(c) I_3 which is a **model** of $\varphi_1 \wedge \varphi_2 \wedge \varphi_3$.

Exercise 4: Truth Value

(1+1+1 Points)

Determine the truth value of the statement $\exists x \forall y (x \leq y^2)$ if the domain (or universe) for the variables consists of:

(a) the positive real numbers,

(b) the integers,

(c) the nonzero real numbers.