Theoretical Computer Science - Bridging Course
Winter Term 2020/21
Revision Sheet

Exercise 1: The class \( P \)

1. Which of the following statements about the class \( P \) are true?
   
   - \( P \) is the class of all languages that are decidable by deterministic multi-tape Turing machines running in polynomial time.
   - A language \( L \) belongs to \( P \) iff there is a constant \( k \) and a decider \( M \) running in time \( O(n^k) \) such that \( L = L(M) \).
   - A language \( L \) belongs to \( P \) iff \( L \) is decided by an \( O(2^n) \) time DTM.
   - \( A_{TM} \) belongs to \( P \).

2. Show that the following language (\( \equiv \) decision problem)
   
   \( 18\text{-DominatingSet} := \{\langle G \rangle \mid G \text{ has a dominating set of size at most 18} \} \)

   is in the class \( P \).

   **Remark:** A subset of the nodes of a graph \( G \) is a dominating set if every other node of \( G \) is adjacent to some node in the subset.

   - Is \( 18\text{-DominatingSet} \) decidable?

Exercise 2: The class \( \mathcal{NP} \) and \( \mathcal{NPC} \)

1. Is the following true: A language \( L \) is decidable by an \( O(\log n) \) time deterministic single tape TM, then \( L \) belongs to \( \mathcal{NP} \).

2. Given a set \( U \) of \( n \) elements (‘universe’) and a collection \( S \subseteq \mathcal{P}(U) \) of subsets of \( U \), a selection \( C_1, \ldots, C_k \in S \) of \( k \) sets is called a set cover of \( (U, S) \) of size \( k \) if \( C_1 \cup \ldots \cup C_k = U \).

   Show that the problem
   
   \( \text{SetCover} := \{\langle U, S, k \rangle \mid U \text{ is a set, } S \subseteq \mathcal{P}(U) \text{ and there is a set cover of } (U, S) \text{ of size } k \} \)

   is \( \mathcal{NP} \)-complete.

   You may use that

   \( \text{DominatingSet} = \{\langle G, k \rangle \mid G \text{ has a dominating set with } k \text{ nodes} \} \)

   is \( \mathcal{NP} \)-complete.

3. Why can’t we solve \( \text{DominatingSet} \) in polynomial time the same way we solve \( 18\text{-DominatingSet} \)?
Exercise 3: Regular and Context Free Languages

- Is the language $L := \{ w \in \text{DOMINATINGSET} \mid |w| \leq 2021 \}$ regular? Is it decidable?
- Using the pumping lemma for regular languages, show that the following language $L_1 = \{ 0^m \mid m \text{ is a prime} \}$ is not regular.
- Using the Pumping Lemma for CFL, show also that $L_1$ is not a CFL.