Albert-Ludwigs-Universität, Inst. für Informatik
Prof. Dr. Fabian Kuhn
Salwa Faour

Theoretical Computer Science - Bridging Course
Winter Term 2020/21
Revision Sheet

Exercise 1: The class $\mathcal{P}$

1. Which of the following statements about the class $\mathcal{P}$ are true?

   - $\mathcal{P}$ is the class of all languages that are decidable by deterministic multi-tape Turing machines running in polynomial time.
   - A language L belongs to $\mathcal{P}$ iff there is a constant k and a decider M running in time $O(n^k)$ such that $L = L(M)$.
   - A language L belongs to P iff L is decided by an $O(2^n)$ time DTM .
   - A_{TM} belongs to $\mathcal{P}$.

2. - Show that the following language ($\cong$ decision problem) 18-DominatingSet $:= \{\langle G \rangle \mid G$ has a *dominating set* of size at most $18\}$ is in the class $\mathcal{P}$.
     Remark: A subset of the nodes of a graph G is a *dominating set* if every other node of G is adjacent to some node in the subset.
   - Is 18-DominatingSet decidable?

Exercise 2: The class $\mathcal{NP}$ and $\mathcal{NPC}$

1. Is the following true: A language L is decidable by an $O(\log n)$ time deterministic single tape TM, then L belongs to NP.

2. Given a set U of n elements ('universe') and a collection $S \subseteq \mathcal{P}(U)$ of subsets of U, a selection $C_1, \ldots, C_k \in S$ of k sets is called a *set cover* of (U, S) of size k if $C_1 \cup \ldots \cup C_k = U$.

   Show that the problem

   $$\text{SetCover} := \{\langle U, S, k \rangle \mid U \text{ is a set, } S \subseteq \mathcal{P}(U) \text{ and there is a set cover of } (U, S) \text{ of size } k\}$$

   is NP-complete.

   You may use that

   $$\text{DominatingSet} = \{\langle G, k \rangle \mid G \text{ has a dominating set with } k \text{ nodes}\}.$$

   is NP-complete.

3. Why can't we solve DominatingSet in polynomial time the same way we solve 18-DominatingSet?

Exercise 3: Regular and Context Free Languages

- Is the language $L := \{w \in \textsc{DominatingSet} \mid |w| \leq 2021\}$ regular? Is it decidable?
- Using the pumping lemma for regular languages, show that the following language $L_1 = \{0^m \mid m \text{ is a prime}\}$ is not regular.
- Using the Pumping Lemma for CFL, show also that L_1 is not a CFL.