Theoretical Computer Science - Bridging Course
Winter Term 2020/21
Exercise Sheet 3

for getting feedback submit electronically by 12:15 pm, Monday, November 23, 2020

Exercise 1: Regular Expressions

Consider the following regular expressions. What language do they recognize? Give two strings that are members of the corresponding language and two strings which are not members – a total of four strings for each part. Assume for the first two parts that the alphabet \( \Sigma = \{a, b, c\} \).

(a) \( a^*b^*c^* \)

(b) \( ((a \cup c)^*b(a \cup c)^*b(a \cup c)^*b(a \cup c)^*)^* \)

Give a regular expression for each of the following languages.

(c) \( L_1 \) is the language, over alphabet \( \{a, b\} \), of all strings starting and ending with the same symbol.

(d) \( L_2 \) is the language, over alphabet \( \{0, 1\} \), of all alternating 0 and 1 strings.

Exercise 2: The Pumping Lemma: Sufficiency or Necessity?

Consider the language \( L = \{c^ma^n b^n \mid m, n \geq 0\} \cup \{a, b\}^* \) over the alphabet \( \Sigma = \{a, b, c\} \).

(a) Describe in words (not using the pumping lemma), why \( L \) cannot be a regular language.

(b) Show that the property described in the Pumping Lemma is a necessary condition for regularity but not sufficient for regularity.

Hint: Use \( L \) as counter example, i.e., show that it can be ’pumped’ (in the sense of the pumping lemma), but is still not regular.

Exercise 3: To Be Regular or Not to Be

Let \( \Sigma = \{0, 1\} \), prove the following:

(a) The language \( A = \{0^k w 0^k \mid k \geq 1 \text{ and } w \in \Sigma^* \} \) is regular.

(b) The language \( B = \{0^k 1 w 0^k \mid k \geq 1 \text{ and } w \in \Sigma^* \} \) is not regular.
Exercise 4: NFAs to Regular Expressions (4 Points)

Consider the following NFA:

Give the regular expression defining the language recognized by this NFA by stepwise converting it into an equivalent GNFA with only two nodes.