

# Theoretical Computer Science - Bridging Course

## Winter Term 2020/21

### Exercise Sheet 4

for getting feedback submit electronically by 12:15 p.m., Monday, November 30th, 2020

#### Exercise 1: Context-Free Grammar (2+3 Points)

For each of the following languages, give a context-free grammar to accept the language. The alphabet set is  $\Sigma = \{0, 1\}$ .

- (a)  $\{w \mid w \text{ contains at least three ones}\}$ .
- (b)  $L_1 = \{w\#w' \mid w^R \text{ is a substring of } w', \text{ and } w, w' \in \{0, 1\}^*\}$ .<sup>1</sup>

#### Exercise 2: Chomsky Normal Form (1+4 Points)

Consider the following context-free grammar (CFG):

$$\begin{aligned} S &\rightarrow aSb \mid D \\ D &\rightarrow ccDc \mid \varepsilon \end{aligned}$$

- a) Which language does this grammar define?
- b) Convert this CFG into an equivalent one in Chomsky Normal Form. Give the grammar you obtained after each step of the conversion algorithm.

#### Exercise 3: Constructing Pushdown Automata (4 Points)

Consider the language  $L = \{a^n b^{2m} b a^n \mid m, n > 0\}$  over the alphabet  $\Sigma = \{a, b\}$ . Construct a PDA  $\mathcal{A}$  with  $L(\mathcal{A}) = L$ .

#### Exercise 4: Pumping Lemma for Context-Free Languages (3+3 Points)

Use the pumping lemma to show that the following languages over the alphabet  $\Sigma = \{a, b\}$  are not context free:

- (a)  $\{ww \mid w \in \{a, b\}^*\}$
- (b)  $\{a^n b a^{2n} b a^{3n} \mid n \geq 0\}$

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<sup>1</sup> $w^R$  is achieved by reversing the order of the symbols in  $w$ .