Albert-Ludwigs-Universität, Inst. für Informatik
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Theoretical Computer Science - Bridging Course
Winter Term 2020/21
Exercise Sheet 5

for getting feedback submit electronically by 12:15 pm, Monday, December 7, 2020

Exercise 1: Turing Machines *(3+3 Points)*

(a) Construct a Turing machine that decides on the languages $C_1 = \{a^i b^j c^k | i - j = k \text{ and } i, j, k \geq 1\}$ and $C_2 = \{a^i b^j c^k | i \times j = k \text{ and } i, j, k \geq 1\}$. You do not need to give a formal description of the Turing machines; the high level description is enough.

(c) Consider a push-down automata with two stacks instead of one. Can it be as powerful as a Turing machine? Justify.

Exercise 2: Constructing Turing Machines I *(4+1+2+1 Points)*

Let $\Sigma = \{0, 1\}$. For a string $s = s_1 s_2 \ldots s_n$ with $s_i \in \Sigma$ let $s^R = s_n s_{n-1} \ldots s_1$ be the *reversed* string. *Palindromes* are strings s for which $s = s^R$. Then $L = \{sas^R \mid s \in \Sigma^*, a \in \Sigma \cup \{\varepsilon\}\}$ is the language of all palindromes over Σ.

(a) Give a state diagram of a Turing machine recognizing L.

(b) Give the maximum number (or a close upper bound for the number) of head movements your Turing machine makes until it halts, if started with an input string $s \in \Sigma^*$ of length $|s| = n$ on its tape.

(c) Describe (informally) the behavior of a 2-tape Turing machine which recognizes L and uses significantly fewer head movements on long inputs than your 1-tape Turing machine.

(d) Give the maximum number (or a close upper bound for the number) of head movements your Turing machine makes on any of the two tapes until it halts, if started with an input string $s \in \Sigma^*$ of length $|s| = n$ on the first tape.

Exercise 3: Constructing Turing Machines II *(6 Points)*

Let $L = \{\langle w \rangle, \langle w + 1 \rangle \mid w \in \mathbb{N}\}$, e.g., the word $\langle 6 \rangle, \langle 7 \rangle = 110, 111$ is contained in L. Design a Turing machine which accepts L. You do not need to provide a formal description of the Turing machine but your description has to be detailed enough to explain every possible step of a computation.
Remark: Here $\langle w \rangle$ is the binary encoding of the number w, e.g., the number 6 is going to be the string 110.