Albert-Ludwigs-Universität, Inst. für Informatik
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Theoretical Computer Science - Bridging Course
Winter Term 2020/21
Exercise Sheet 6

for getting feedback submit electronically by 12:15 pm, Monday, December 14th, 2020

Exercise 1: Constructing Turing Machines

Construct a Turing Machine for each of the following languages.

(a) $L_1 = \{a^i b^j a^i b^j | i, j > 0\}$

(b) Language L_2 of all strings over alphabet $\{a, b\}$ with the same number of a's and b's.

Remark: It is sufficient to give a detailed description of the Turing Machines. You do not need to give formal definitions.

Exersive 2: Semi-Decidable vs. Recursively Enumerable

Very often people in computer science use the terms *semi-decidable* and *recursively enumerable* equivalently. The following exercise shows in which way they actually are equivalent. We first recall the definition of both terms.

A language L is *semi-decidable* if there is a Turing machine which accepts every $w \in L$ and does not accept any $w \notin L$ (this means the TM can either reject $w \notin L$ or simply not stop for $w \notin L$).

A language is *recursively enumerable* if there is a Turing machine which eventually outputs every word $w \in L$ and never outputs a word $w \notin L$.

(a) Show that any recursively enumerable language is semi-decidable.

(b) Show that any semi-decidable language is recursively enumerable.

Exercise 3: Decidability

1. The *special halting problem* is defined as

$$H_s = \{\langle M \rangle \mid \langle M \rangle \text{ encodes a TM and } M \text{ halts on } \langle M \rangle\}.$$

   Show that H_s is undecidable.

   Hint: Assume that M is a TM which decides H_s and then construct a TM which halts iff M does not halt. Use this construction to find a contradiction.

2. Show that $A = \{\langle R, S \rangle \mid R \text{ and } S \text{ are regular expressions and } L(R) \subseteq L(S)\}$ is decidable.

3. Show that $EQ_{TM} = \{\langle M_1, M_2 \rangle \mid M_1, M_2 \text{ are Turing Machines and } L(M_1) = L(M_2)\}$ is undecidable.

   Hint: You may use that $E_{TM} = \{\langle M \rangle \mid M \text{ is a Turing Machine and } L(M) = \emptyset\}$ is undecidable.