Exercise 1: Another Decidability Question

Let \( S = \{ \langle M \rangle \mid M \text{ is a DFA that accepts } w^R \text{ whenever it accepts } w \} \). Show that \( S \) is a decidable language.

Remark: You can use that it is not difficult to construct a TM, which tests whether an input is the well formed encoding of a DFA.

Exercise 2: Big-O Notation

The set \( \mathcal{O}(f) \) contains all functions that are asymptotically not growing faster than the function \( f \) (when additive or multiplicative constants are neglected). That is:

\[
g \in \mathcal{O}(f) \iff \exists c \geq 0, \exists M \in \mathbb{N}, \forall n \geq M : g(n) \leq c \cdot f(n)
\]

For the following pairs of functions, check whether \( f \in \mathcal{O}(g) \) or \( g \in \mathcal{O}(f) \) or both. Prove your claims (you do not have to prove a negative result \( \notin \), though).

(a) \( f(n) = 20n, \ g(n) = 0.1 \cdot n^2 \)
(b) \( f(n) = \sqrt{n^2}, \ g(n) = \sqrt{n} \)
(c) \( f(n) = \log_2(2^n \cdot n^3), \ g(n) = 3n \) \quad \text{Hint: You may use that } \log_2 n \leq n \text{ for all } n \in \mathbb{N}.

Exercise 3: Sorting Functions by Asymptotic Growth

Sort the following functions by asymptotic growth using the \( \mathcal{O} \)-notation. Write \( g <_\mathcal{O} f \) if \( g \in \mathcal{O}(f) \) and \( f \notin \mathcal{O}(g) \). Write \( g =_\mathcal{O} f \) if \( f \in \mathcal{O}(g) \) and \( g \in \mathcal{O}(f) \).

\[
\begin{array}{cccc}
n^2 & \sqrt{n} & 2^n & \log(n^2) \\
3^n & n^{100} & \log(\sqrt{n}) & (\log n)^2 \\
\log n & 10^{100}n & n! & n \log n \\
n \cdot 2^n & n^t & \sqrt{\log n} & n
\end{array}
\]