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Algorithms and Data Structures
Winter Term 2021/2022
Exercise Sheet 4

Exercise 1: Hashing - Collision Resolution with Open Addressing

(a) Let $h(s, j) := h_1(s) - 2j \bmod m$ and let $h_1(x) = x + 2 \bmod m$. Insert the keys 51, 13, 21, 30, 23, 72 into the hash table of size $m = 7$ using linear probing for collision resolution (the table should show the final state).

0	1	2	3	4	5	6

(b) Let $h(s, j) := h_1(s) + j \cdot h_2(s) \bmod m$ and let $h_1(x) = x \bmod m$ and $h_2(x) = 1 + \big(x \bmod (m-1)\big)$. Insert the keys 28, 59, 47, 13, 39, 69, 12 into the hash table of size $m = 11$ using the double hashing probing technique for collision resolution. The hash table below should show the final state.

0	1	2	3	4	5	6	7	8	9	10

Exercise 2: Application of Hashtables

Consider the following algorithm:

Algorithm 1 `algorithm` ▷ Input: Array A of length n with integer entries

for $i = 1$ to $n - 1$ **do**
  for $j = 0$ to $i - 1$ **do**
    for $k = 0$ to $n - 1$ **do**
      if $|A[i] - A[j]| = A[k]$ **then**
        return true
return false

(a) Describe what `algorithm` computes and analyse its asymptotical runtime.

(b) Describe a different algorithm $\mathcal{B}$ for this problem (i.e., $\mathcal{B}(A) =$ `algorithm`(A) for each input A) which uses hashing and takes time $\mathcal{O}(n^2)$.

You may assume that inserting and finding keys in a hash table needs $\mathcal{O}(1)$ if $\alpha = \mathcal{O}(1)$ (α is the load of the table).

(c) Describe another algorithm for this problem without using hashing which takes time $\mathcal{O}(n^2 \log n)$.