Exercise 1: Binary Search Trees I

Consider the following binary search tree.

![Binary Search Tree Diagram]

1. Give all sequences of insert(key) operations that generate the tree.

2. Draw the tree after the following sequence of operations: insert(6), insert(5), remove(3).

Exercise 2: Binary Search Trees II

(a) Describe a function that takes a binary search tree \( B \) and a key \( x \) as input and generates the following output:

- If there is an element \( v \) in \( B \) with \( v.key = x \), return \( v \).
- Otherwise, return the pair \((u, w)\) where \( u \) is the tree element with the next smaller key and \( w \) is the element with the next larger key. It should be \( u = \text{None} \) if \( x \) is smaller than any key in the tree and \( w = \text{None} \) if \( x \) is larger than any key in the tree.

For your description you can use pseudo code or a sufficiently detailed description in English.

Analyze the runtime of your function.

(b) Describe a function which returns the depth of a binary search tree and analyze the runtime.

(c) Describe a function that for a given binary search tree with \( n \) nodes and a given \( k \leq n \) returns a list with the \( k \) smallest keys from the tree. Analyze the runtime.