

Algorithms and Data Structures

Winter Term 2021/2022

Exercise Sheet 11

Exercise 1: Bitstrings without consecutive ones

Given a positive integer n , we want to compute the number of n -digit bitstrings without consecutive ones (e.g., for $n = 3$ this number is 5, as 000, 001, 010, 100, 101 are the 3-digit bitstrings without consecutive ones).

- (a) Give an algorithm which solves this problem in time $\mathcal{O}(n)$. Explain the runtime.
- (b) Implement your solution. You may use the template `DP.py`.

Exercise 2: Partitioning

Given a set $X = \{x_0, \dots, x_{n-1}\}$ with $x_i \in \mathbb{N}$, we want to determine whether *there is* a subset $S \subseteq X$ such that $\sum_{x \in S} x = \sum_{x \in X \setminus S} x$ (it is not necessary to compute S).

- (a) Let $W := \sum_{x \in X} x$. Give a recursive formula $s : \{0, \dots, n-1\} \times \{0, \dots, W\} \rightarrow \{\text{True}, \text{False}\}$ such that $s(i, j) = \text{True}$ if and only if there is a $S \subseteq \{x_0, \dots, x_i\}$ such that $\sum_{x \in S} x = j$. Explain how s can be used to solve the above problem in time $\mathcal{O}(W \cdot n)$.
- (b) Implement your solution. You may use the template `DP.py`. Run your algorithm on the sets given in `set1.txt`, `set2.txt` and `set3.txt`.