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Algorithm Theory
Exercise Sheet 1

Due: Tuesday, 26th of October, 2021, 4 pm

Exercise 1: Sort Functions by Asymptotic Growth *(8 Points)*

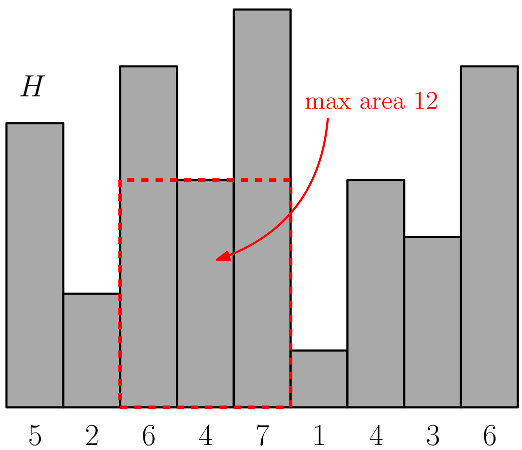
Give a sequence of the following functions sorted by asymptotic growth, i.e., for consecutive functions g, f in your sequence, it should hold $g \in \mathcal{O}(f)$. Write "$g \cong f$" between two functions in the sequence if $f \in \mathcal{O}(g)$ *and* $g \in \mathcal{O}(f)$.

Note: *No formal proofs required, but you loose* 1 *point for each error.*

$\log_2(n!)$	$\sqrt{n}$	$\sqrt{n^3}$	$\log_2(n^2)$
$n^2 + n\log_2(n^2)$	3^n	$n^{\log_2 n}$	$(\log_2 n)^2$
$\log_{10} n$	$10^{100} \cdot n$	$n!$	$n \log_2 n$
$n \cdot 2^n$	n^n	$\sqrt{\log_2 n}$	n^2

Exercise 2: Maximum Rectangle in a Histogram *(12 Points)*

Consider a sequence $h_1, \dots, h_n$ of positive, integer numbers. This sequence represents a histogram H consisting of n horizontally aligned bars each of width 1, where h_i represents the height of the i^{th} bar. The goal is to find a rectangle of maximum area completely within H (i.e., within the union of bars).



(a) Describe an algorithm that computes a maximum area rectangle in H in time $\mathcal{O}(n^2)$.

(b) Describe an algorithm that computes a maximum area rectangle that is within H and also intersects the i^{th} bar in time $\mathcal{O}(n)$ and prove the running time.

Remark: correct solutions in $o(n^2)$ grant partial points.

(c) Give an algorithm that uses the divide and conquer principle to compute a maximum area rectangle in H in time $\mathcal{O}(n \log n)$ and prove the running time.

Remark: you can use part (b), even if you did not succeed.