



Algorithm Theory

Exercise Sheet 8

Due: Tuesday, 14th of December, 2021, 4 pm

Exercise 1: Work Schedule

(10 Points)

Assume you want to design a work schedule for a hospital for the next n days. The hospital employs k doctors. On day i , exactly p_i doctors need to be present, for $i = 1, \dots, n$. Each doctor j provides a set $L_j \subseteq \{1, \dots, n\}$ of days on which he or she is willing to work.

- (a) Describe a polynomial-time algorithm that either
- returns a list $L'_j \subseteq L_j$ of working days for each doctor j such that on day i , exactly p_i doctors are present or
 - reports that there is no such set of lists that fulfills the given constraints.
- (b) The hospital finds that the doctors tend to submit lists that are much too restrictive, and so it often happens that there is no feasible working schedule. Thus, the hospital relaxes the requirements in the following way. For some number $c > 0$, each doctor j can be forced to work on c days which are not in his/her list L_j .

Give a polynomial-time algorithm to solve this problem, i.e., the algorithm should either

- return a list L'_j of working days for each doctor j with $|L'_j \setminus L_j| \leq c$ such that on day i , exactly p_i doctors are present or
- report that there is no such set of lists that fulfills the given constraints.

Exercise 2: Minimum Cut with Maximum Edges

(10 Points)

Let $G = (V, E, c)$ be a flow network with positive integer capacities and $s, t \in V$. Give an efficient algorithm to find a minimum s - t cut with the largest number of edges. That is, we want to find an s - t cut (A, B) which

- is minimum (w.r.t. the edge weights)
- among all minimum s - t cuts, has a maximum number of edges going from A to B .

Show that your algorithm is correct and analyze its runtime as a function of $|E|$ and C (minimum s - t cut capacity of G).