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Algorithm Theory
Exercise Sheet 9

Due: Tuesday, 21st of December, 2021, 4 pm

Exercise 1: Hypergraph Matching *(10 Points)*

A *hypergraph* is a generalization of an undirected graph in which edges consist of arbitrary subsets of vertices. That is, a hypergraph consists of a set of nodes V and a set of edges $E \subseteq \mathcal{P}(V)$. A hypergraph is called k-uniform if each edge has size k, i.e., contains exactly k nodes. For example, a simple, undirected graph is a 2-uniform hypergraph.
A matching of a hypergraph is a set of edges which are pairwise disjoint. The definitions of a maximal and maximum matching are extended to hypergraphs in the obvious way.

(i) Show that a maximum matching of a k-uniform hypergraph has size at most $\left\lfloor \frac{n}{k} \right\rfloor$.

(ii) For an arbitrary $k \geq 2$, provide a k-uniform hypergraph G and a maximal matching M such that $|M| = \frac{|M^*|}{k}$ where M^* is a maximum matching.

(iii) Show that for any maximal matching M of a k-uniform hypergraph we have $|M| \geq \frac{|M^*|}{k}$ where M^* is a maximum matching.

Exercise 2: Randomization *(10 Points)*

Assume you are given a randomized algorithm $\mathcal{A}$ that given a graph G with n nodes and a maximum matching of size s, computes an integer $k \leq s$ in time $T(n)$ such that with probability at least $p(n)$ we have $k = s$.
Give an algorithm with running time $\mathcal{O}\big(p(n)^{-1} \cdot T(n) \cdot \log n\big)$ that computes the size of a maximum matching of a graph with n nodes with probability at least $1 - \frac{1}{n}$. Prove the success probability.