University of Freiburg
Dept. of Computer Science
Prof. Dr. F. Kuhn
P. Bamberger, P. Schneider

Algorithm Theory
Exercise Sheet 12

Due: Tuesday, 25th of January, 2022, 4 pm

Exercise 1: Minimum Vertex Cover Approximation *(20 Points)*

Let $G = (V, E)$ be an *undirected, unweighted* graph. Consider the following algorithms that give approximate solutions to the minimum vertex cover problem.

Algorithm 1 `alg1`

1: $S \leftarrow \emptyset$ ▷ create an empty set
2: **while** $E \neq \emptyset$ **do**
3: pick some edge $\{u, v\} \in E$
4: $S \leftarrow S \cup \{u, v\}$
5: remove edges incident to u or v from E
6: **return** S

Algorithm 2 `alg2`

1: $S \leftarrow \emptyset$ ▷ create an empty set
2: **while** $E \neq \emptyset$ **do**
3: pick vertex $v \in V$ of maximal degree
4: $S \leftarrow S \cup \{v\}$
5: remove edges incident to v from E
6: **return** S

(a) Show that `alg1` and `alg2` output valid vertex covers. *(3 Points)*

(b) Argue why `alg1` provides a 2-approximation of the minimum vertex cover problem. *(3 Points)*

Hint: You can use results that we proved in the lecture.

(c) Argue why `alg2` provides a $O(\log n)$-approximation of the minimum vertex cover problem. *(5 Points)*

Hint: You can use results that we proved in the lecture.

(d) Show that the solution provided by `alg2` is only a $\Theta(\log n)$ approximation for some graphs. *(9 Points)*

Hint: Give a bipartite graph with node set $V = L \cup R$ and $|L| = k$ and $|R| = \Theta(k \log k)$, where `alg2` outputs R but the best solution would be L.