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Algorithm Theory
Exercise Sheet 13 - Bonus*

Due: Tuesday, 1st of February, 2021, 4 pm

**Bonus points can be earned normally, but do not increase the* threshold *for the "Studienleistung".*

Exercise 1: Online Vertex Cover *(10* Points)*

Let $G = (V, E)$ be an unweighted undirected graph. Consider the following online version of the *minimum vertex cover* problem. Initially, we are given the set of nodes V and an empty vertex cover $S = \emptyset$. Then, the edges appear one-by-one in an online fashion. When a new edge $\{u, v\}$ appears, the algorithm needs to guarantee that the edge is covered (i.e., if this is not already the case, at least one of the two nodes u and v needs to be added to S). Once a node is in S it cannot be removed from S.

(a) Provide a deterministic online algorithm with strict competitive ratio at most 2. *(1 Point)*

(b) Show that any deterministic online algorithm for the online vertex cover problem has strict competitive ratio at least 2. *(4 Points)*

(c) Use Yao's principle to show that any randomized online algorithm for the online vertex cover problem has a strict competitive ratio at least 3/2. *(5 Points)*

Exercise 2: Maximum Cut *(10* Points)*

Let $G = (V, E)$ be an unweighted undirected graph. A *maximum* cut of G is a cut whose size is at least the size of any other cut in G. We consider an online version of the maximum cut problem, where the nodes V of a graph $G = (V, E)$ appear in an online fashion. The algorithm should partition the nodes V into two sets A and B such that induced cut is as large as possible. Whenever a new node appears we also learn its edges to the nodes that have *already appeared before*. An incoming node has to be immediately assigned to either A or B and that decision is final.

(a) Describe a deterministic online maximum cut algorithm with *strict competitive ratio* at least 1/2.

Hint: An online algorithm for a maximization problem is said to have strict competitive ratio α if it guarantees that $\text{ALG} \geq \alpha \cdot \text{OPT}$*, where* ALG *and* OPT *are the solutions of the online algorithm and of an optimal offline algorithm, respectively.* *(5 Points)*

(b) Show that no deterministic online algorithm for the online maximum cut problem can have a (constant) strict competitive ratio that is better (larger) than 1/2. *(5 Points)*