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Theoretical Computer Science - Bridging Course
Exercise Sheet 2

Due: Sunday, 7th of November 2021, 23:59 pm

Exercise 1: DFAs under Construction   *(2+2+2 Points)*

Construct DFAs that recognize the following languages. The alphabet set is $\Sigma = \{a, b\}$.

1. $L_0 = \{w \mid w \text{ ends with } ab\}$.

2. $L_1 = \{w \mid w \text{ is any string except } bb \text{ and } bbb\}$.

3. $L_2 = \{w \mid w \text{ contains at least two } a\text{'s and at most one } b\}$.

Bonus part: Construct DFAs that recognize $L_2 \backslash L_1 = \{w \mid w \in L_2 \text{ and } w \notin L_1\}$ and $L_1 \cup L_2$.

Exercise 2: Closure Time   *(2+3+2+2 Points)*

1. Show that if M is a DFA that recognizes language L, you can construct a new DFA M' that recognizes the complement of L i.e. $\bar{L} := \Sigma^* \setminus L$. Conclude that the class of regular languages is closed under complementation.

2. Show by giving an example that if M is an NFA (instead of a DFA) that recognizes language L, then the same approach you used to construct the new DFA M' above doesn't necessarily yield a new NFA that recognizes the complement of L. Is the class of languages recognized by NFAs closed under complementation? Explain your answer.

   Let L_1 and L_2 be regular languages.

3. Show that $L_1 \cap L_2$ is regular by constructing its corresponding DFA.

4. Deduce from parts 1 and 3 that regular languages are closed under the symmetric difference i.e. $L_1 \Delta L_2$ is also regular.

Remark: For parts 1 and 3 there's no need for drawing state diagrams. Show how a DFA for the language in the question can be constructed presuming the existence of DFAs for $L, L_1,$ and L_2.

Exercise 3: From NFA to DFA   *(2+3 Points)*

Consider the following NFA.

b

a, b

start $\longrightarrow$ q_0 q_1

b

b

b

q_2
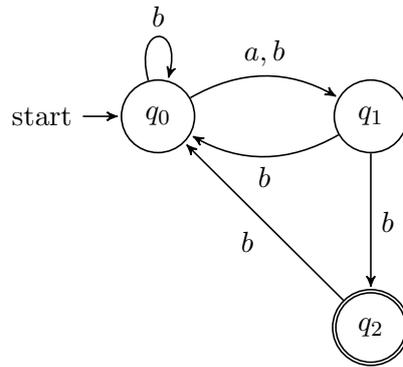
(a) Give a formal description of the NFA by giving the alphabet, state set, transition function, start state and the set of accept states.

(b) Construct a DFA which is equivalent to the above NFA by drawing the corresponding state diagram.

Bonus question: Explain which language the automaton accepts.