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Theoretical Computer Science - Bridging Course
Exercise Sheet 4

Due: Sunday, 21st of November 2021, 23:59 pm

Exercise 1: Context Free Grammar *(1+2+1+2 Points)*

Give a context free grammar for each of the following languages, where the alphabet set is $\Sigma = \{a, b\}$.

(a) $L_1 = \{a^k b^{2k} | k > 0\}$

(b) $L_2 = \{a^i b^j | 0 < i \leq j\}$

(c) $L_1 \cup L_2$

(d) $L_3 = \{w\#w' | w^R \text{ is a substring of } w', \text{ and } w, w' \in \{a, b\}^*\}$ [1]

Exercise 2: Chomsky Normal Form *(4 Points)*

Convert the following CFG into an equivalent CFG in Chomsky Normal Form (CNF). Write down the grammar you obtain after each step of the conversion algorithm.

$$\begin{aligned} A &\rightarrow BAB \mid B \mid \epsilon \\ B &\rightarrow 00 \mid \epsilon \end{aligned}$$

Exercise 3: Constructing Pushdown Automata *(4 Points)*

Consider the language $L = \{a^n b^{2m} b a^n \mid m, n > 0\}$ over the alphabet $\Sigma = \{a, b\}$.
Construct a PDA $\mathcal{A}$ with $L(\mathcal{A}) = L$.

Exercise 4: Pumping Lemma for Context-Free Languages *(3+3 Points)*

Use the pumping lemma to show that the following languages over the alphabet $\Sigma = \{a, b\}$ are not context free:

(a) $L_1 = \{a^m \mid m \text{ is a prime}\}$

(b) $L_2 = \{a^n b a^{2n} b a^{3n} \mid n \geq 0\}$

[1] w^R is achieved by reversing the order of the symbols in w.