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Combinatorial Optimization

Exercise 1 (Optimizing vs. Finding Feasible Solutions)

(1) Consider the LPs

$$\max\{c^\top x : A'x \leq b', A''x \leq b'', x \geq 0\} \tag{*}$$

where $b' \geq 0$ and $b'' < 0$ and

$$\min\{(1^\top A'')x + 1^\top y : A'x \leq b', A''x + y \geq b'', x, y \geq 0\}. \tag{**}$$

from the lecture. Observe that $(x, y)^\top = (0, 0)^\top$ is a feasible vertex for (**) and we may hence run Simplex on that LP. Show that the LP (*) is feasible if and only if the optimum value for (**) is exactly $1^\top b''$.

This shows that if we are able to optimize (with a given initial vertex) then we can decide if a certain LP of interest is feasible.

(2) Now show that the converse is also true. Suppose that you are interested in the LP

$$\max\{c^\top x : Ax \leq b\}. \tag{***}$$

Assume that you can decide if any given LP is feasible and derive a feasible solution, if so. Use this knowledge to show that you can derive the optimum solution of (***) or infer that it is infeasible.

Exercise 2 (Vertices with Capacities)

Recall the definition of networks from the lecture and note that only the edges (but not the vertices) have capacities. Now we want to extend this. Let $d : V \to \mathbb{R}^+$ be a function, called the *capacity* of a vertex and we define that a function $f : E \to \mathbb{R}^+$ is a *feasible* flow if it satisfies the flow-conditions from the lecture and has the additional property

$$\sum_{e \in \delta^-(v)} f(e) \leq d(v)$$

for all vertices v. That is, the incoming flow does not exceed the vertex-capacity.
Formulate the maximum flow problem with vertex-capacities as an ordinary maximum flow problem.

Exercise 3 (Isolate the Commander)

A commander is located at a vertex in an undirected communication network and his soldiers are located at vertices denoted by a set S. Let u_{ij} be the effort required to eliminate the edge ij from the network. How can you determine the minimal effort needed so that the commander can not communicate with any of his soldiers?

Exercise 4 (Guest Shuffle)

You are organizing a dinner and lay n tables. You invite m families to join the dinner and family i has a_i members. Furthermore table j has b_j seats. In order to boost the inter-family-communication you want to make sure that no two members of the same family are at the same table (if this is possible). Formulate this seating arrangement problem as a maximum flow problem.