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Combinatorial Optimization

Exercise 1 (Set Cover)

Construct an instance for which the approximation-factor of H_n for the Greedy algorithm for Set Cover is tight.

Exercise 2 (Set Multicover)

The Set Multicover problem is a generalization of Set Cover, where each element e needs to be covered by a specified integer number r_e of times. The objective is to cover all elements up to their coverage requirements at minimum cost. You can assume that the cost of picking a set S k times is $kc(S)$.
Extend the Greedy algorithm for Set Cover to handle this problem. Show that it is a H_n-approximation algorithm.
Hint. One way to do it is to apply the dual fitting method.

Exercise 3 (Maximum Coverage)

The Maximum Coverage problem is the following: We are given a universe U of n elements with non-negative weights specified, a collection of subsets of U, $S_1, \ldots, S_\ell$ and an integer k. Our task is to pick k sets as to maximize the total weight of the elements covered.
Show that the obvious algorithm of greedily picking the best set in each iteration until k sets are picked achieves an approximation factor of

$$1 - \left(1 - \frac{1}{k}\right)^k > 1 - \frac{1}{e}.$$