Algorithm 5.1 Simple Rounding Set Cover

Input. Universe U with n elements, collection $\mathcal{S} = \{S_1, \ldots, S_k\}$, $S_i \subseteq U$, a cost function $c : \mathcal{S} \to \mathbb{R}$.

Output. Vector $x \in \{0,1\}^k$

Step 1. Set $x = 0$, solve the LP relaxation below, and call the optimal solution z.

$$\begin{aligned} \text{minimize} \quad & \text{val}(x) = \sum_{j=1}^{n} c(S) x_S, \\ \text{subject to} \quad & \sum_{S:e\in S} x_S \geq 1 \quad e \in U, \\ & x_S \geq 0 \quad S \in \mathcal{S}. \end{aligned}$$

Step 2. For each set S set $x_S = 1$ if $z_S \geq 1/f$.

Step 3. Return x.