Winter Term 2001/02
December 17, 2001

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Algorithms Theory

Problem Set 5

To be submitted on January 7, 2002 (in the lecture)

Problem 1: (10 points)
In the lecture we studied perfect hashing. We showed that a perfect hash function and an associated hash table can be constructed in expected time $O(n)$ using space $O(n)$. Is it possible to reduce the space requirement at the expense of increasing the time of constructing the table? Hint: Change the constants in the corollaries.

Problem 2: (10 points)
Let $H_n = \sum_{k=1}^{n} 1/k$. Prove that $\ln(n+1) \leq H_n \leq \ln n + 1$.

Problem 3: (10 points)
Consider a treap T with n elements whose keys are $1, \ldots, n$. Determine the expected number of elements in the subtree of the node containing the element with key k. (The subtree of a node v is the set of nodes that can be reached from v on a descending path of length at least 0.)

Problem 4: (10 points)
In the lecture we studied skip list where the height of each element is determined as follows. Flip a fair coin; if heads come up for the first time in coin flip h, then the height of the element is h.
Consider the modification where we use a coin that, in any trial, comes up heads with probability p, $0 < p < 1$. Determine as exactly as possible the expected time of an unsuccessful search operation.