Winter Term 2001/02
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Algorithms Theory

Problem Set 6

To be submitted on January 21, 2002 (in the lecture)

Problem 1: (10 points)
Show how to modify the pattern matching algorithm by Knuth, Morris and Pratt so as to find *every* occurrence of a pattern P that occurs as a substring in T, while still running in $O(n+m)$ time.

Problem 2: (10 points)
The pattern matching algorithm by Knuth, Morris and Pratt (KMP) can be modified to run faster on binary strings by redefining the prefix function as

$$\pi(j) = \text{the largest } k < j \text{ such that } P[1..k-1]\overline{p}_k \text{ is a suffix of } P[2..j],$$

where $\overline{p}_k$ is the complement of the kth bit of P. Modify the KMP algorithm to be able to take advantage of this new prefix function. Show that this method makes at most n comparisons between the text and the pattern, as opposed to the $2n$ comparisons needed by the standard KMP algorithm.

Problem 3: (3 + 7 points)

a) Let $w = e^{2\pi i/n}$. Show that if n is even, then $w^{n/2} = -1$.

b) Consider the polynomial $p(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + a_5x^5 + a_6x^6 + a_7x^7$. Describe how the FFT algorithm evaluates $p(w^l)$, for any $0 \leq l \leq 7$, i.e. identify the polynomials that are evaluated in each of the recursive levels.

Problem 4: (3 + 3 + 4 points)
Determine as exactly as possible the solution of the following recurrences. Show that your solution is correct. In parts b) and c) you can assume that n is a power of 2.

a) $T(1) = 1$ and $T(n) = T(n-1) + 2n$, for $n \geq 2$.

b) $T(1) = 1$ and $T(n) = 2T(n/2) + 1$ for $n \geq 2$.

c) $T(1) = 1$ and $T(n) = 2T(n/2) + cn$ for $n \geq 2$ and some positive constant c.