Winter Term 2001/02 | Prof. Dr. Susanne Albers
January 21, 2002 | Institute of Computer Science

Algorithms Theory

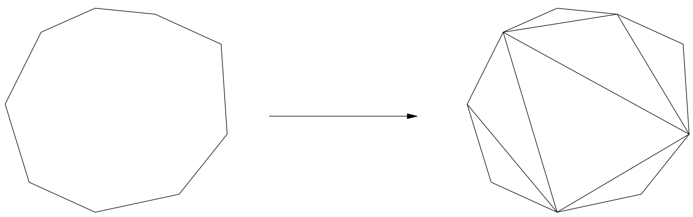
Problem Set 7

To be submitted on February 4, 2002 (in the lecture)

Problem 1: (10 points)
Consider the currency problem with coin denominations $c_1 = 1$, $c_2 = 2$, $c_3 = 5$, $c_4 = 10$ and $c_5 = 25$. Show that the greedy algorithm presented in the lecture uses the minimum possible number of coins to pay an amount of N, for any $N \in \mathbb{N}$.

Problem 2: (10 points)
Let $I = (a_1, \ldots, a_n)$ be an input for the bin packing problem. The item sizes are in non-increasing order, i.e. $1 \geq a_1 \geq a_2 \geq \ldots \geq a_n > 0$. Let $\mathrm{OPT}(I)$ be the optimal (minimum) number of bins for I. Show that all the items placed by FIRST-FIT-Decreasing in extra bins, i.e. in bins with index larger then $\mathrm{OPT}(I)$, have size at most 1/3.

Problem 3: (5 + 5 points)
In this problem we consider *triangulations* of convex polygons. A *simple polygon* is a piecewise-linear, closed curve in the plane that does not cross itself. A simple polygon is *convex* if, given any two points on its boundary or in its interior, all points on the line segment drawn between them are contained in the polygon's boundary or interior.
We can represent a convex polygon P by listing its vertices in counterclockwise order starting at an arbitrary vertex v_0, i.e. $P = (v_0, \ldots, v_{n-1})$. Given two non-adjacent vertices v_i and v_j, the segment $\overline{v_i v_j}$ is a *chord* in the polygon. A chord $\overline{v_i v_j}$ divides the polygon into two convex polygons $(v_i, v_{i+1}, \ldots, v_j)$ and $(v_j, v_{j+1}, \ldots, v_i)$, where indices are taken $\bmod n$. A *triangulation* of a polygon is a set T of chords that divides the polygon into disjoint triangles. In a triangulation, no chords intersect. The figure below shows an example.



a) Define a weight function w on triangles with vertices v_i, v_j, v_k as $w(v_i v_j v_k) = |v_i v_j| + |v_j v_k| + |v_k v_i|$, where $|v_i v_j|$ is the Euclidian distance between v_i and v_j. The goal is to find a triangulation that minimizes the sum of the weights of the triangles. Solve this problem using dynamic programming.

b) Prove that every triangulation of an n-vertex polygon has $n - 3$ chords and divides the polygon into $n - 2$ triangles.

Problem 4: (10 points)

Prove that the online paging algorithm FIFO (First-In First-Out) is k-competitive, where k is the number of pages that can be stored in fast memory.