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Institut für Informatik
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Fourth Assignment

Selected Topics in Efficient Algorithms

To be returned in the lectures on December 18th, 2007.

Exercise 1: Prove Theorem 3.1 from the lectures.

Exercise 2: Show that a graph G is bipartite if and only if every cycle in it has even length. Give a linear time algorithm that decides if G is bipartite and prove its correctness.

Exercise 3: Given any graph G, we define the *block-cutvertex graph* $bc(G)$ as follows. The vertices of $bc(G)$ are the blocks and the cut vertices of G; a block B and a cut vertex c of G are adjacent in $bc(G)$ if and only if c is contained in B. Show that the following assertions hold.

1. If G is connected, $bc(G)$ is a tree.
2. For each vertex v of G, let $b(v)$ denote the number of blocks containing v. Moreover, let $b(G)$ be the number of blocks of G, and denote the number of connected components of G by p. Then
$$b(G) = p + \sum_v (b(v) - 1).$$
3. For each block B, let $c(B)$ be the number of cut vertices contained in B, and let $c(G)$ be the number of all cut vertices of G. Then
$$c(G) = p + \sum_B (c(B) - 1).$$
4. $b(G) \geq c(G) + 1$.

Exercise 4: Let $G = (V, E)$ be a graph. A trail w in G is called *hamiltonian* if w enters every vertex $v \in V$ exactly once. A graph is called *hamiltonian connected* if for every pair of vertices $v, w \in V$ $(v \neq w)$ there exists a hamiltonian trail in G from v to w. Show that for every $n \in \mathbb{N}$ there is a graph G_n over n vertices and at most $2n - 2$ edges which is hamiltonian connected.

Exercise 5: A topological order is a numbering of the vertices of a directed acyclic graph (DAG) such that if $(u, v) \in E$ than the number of u is smaller than the number of v. Give a linear time algorithm that returns a topological order on a given DAG G.

Please visit the following link frequently for ongoing information:
`http://www.informatik.uni-freiburg.de/~ipr/ipr_teaching/ws07_08/selected_topics.html`