Albert-Ludwigs-Universität Freiburg
Institut für Informatik
Prof. Dr. Susanne Albers
Dr. Alexander Souza

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Sheet 03

Average-Case Analysis

Exercise 1 (Live and Let Die)

You are a prisoner sentenced to death. The Emperor offers you a chance to live by playing a simple game. He gives you 50 black marbles, 50 white marbles and 2 empty bowls. He then says, "Divide these 100 marbles into these 2 bowls. You can divide them any way you like as long as you use all the marbles. Then I will blindfold you and mix the bowls around. You then can choose one bowl and remove *one* marble. If the marble is *white* you will live, but if the marble is *black* you will die."
How do you divide the marbles up so that you have the greatest probability of choosing a white marble?

Exercise 2 (Running Time of Nemhauser-Ullmann)

Give an instance showing that the Nemhauser-Ullmann algorithm for Knapsack requires exponential running time.

Exercise 3 (Implementing Nemhauser-Ullmann)

Show that the computation of the sets S_j in the Nemhauser-Ullmann algorithm can be done in time $O(q_j)$, where the q_j are arbitrary upper bounds on $|S_j|$.

Exercise 4 (Stochastic Weights)

Recall the notation of Knapsack with stochastic weights and prove: For any $x \in \{0,1\}^n$ we have $\Pr[\text{weight}(x) \leq 1] \geq 1 - \text{mass}(x)$.