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Sheet 04

Average-Case Analysis

Exercise 1 (Kill Bill)

In the movie “Kill Bill – Volume 1”, Beatrix Kiddo (Uma Thurman) defeats an opponent by fragmenting his sword into three pieces with her Hattori Hanzo Samurai sword. Assuming that the blade originally had length ℓ and that the two cuts are uniform distributed over it, what is the probability that the three pieces can from a triangle.

Exercise 2 (Expectations of Expectations)

Prove that for any random variables X and Y it holds that

$$\mathbb{E}\left[X\right] = \mathbb{E}\left[\mathbb{E}\left[X \mid Y\right]\right].$$

(This formula may look strange at first sight, but notice that $\mathbb{E}\left[X \mid Y\right]$ *is* a random variable, since Y is one.)

Exercise 3 (Completion Time Scheduling)

Recall the notation of Completion Time Scheduling. Show that $\text{val}(\pi) = \text{val}(\pi^*) + \sum_j \sum_{k >^\pi j} \delta_{jk} x_{jk}$, where $\delta_{jk} = w_j p_k - w_k p_j$ and x_{jk} is one if $p_j/w_j > p_k/w_k$ and zero otherwise.

Exercise 4 (Memoryless Property of Geometric Distributions)

Let $h : \mathbb{N}_1 \times \cdots \times \mathbb{N}_n \to \mathbb{N}$ be a monotone non-decreasing function. Let $P = (P_1, \ldots, P_n)$ denote the vector of independent geometric distributed random variables P_j. Show that for any j and any integer $t \geq 0$ it holds that

$$\mathbb{E}\left[\frac{P_j - t}{h(P)} \;\middle|\; P_j > t\right] \leq \mathbb{E}\left[\frac{P_j}{h(P)}\right].$$