
Average-Case Analysis

Exercise 1 (Knights of the Round Table)

King Arthur, Merlin, Sir Lancelot, Sir Gawain, and Guinevere decide to go to their favorite restaurant to share some mead and grilled meats. They sit down at a round table for five, and as soon as they do, Lancelot notes, “We sat down around the table in age order! What are the odds of that?”

Merlin smiles broadly. “This is easily solved without any magic.” He then shared the answer. What did he say the odds were?

Exercise 2 (Paging Lower Bound)

Prove that any deterministic paging algorithm has competitive ratio at least k , where k is the cache size.

Hint. The crucial step of the proof is to show that the (optimal) LONGEST FORWARD DISTANCE algorithm faults at most $|R|/k$ times on any sequence R of requests chosen from a set of $k + 1$ distinct pages.

Exercise 3 (Variable Cache Size)

Suppose that we may choose the cache size $k \in \{a, \dots, b\}$ even after the adversary has committed to a request sequence. The goal of this exercise is to show that this additional freedom does not help.

Show that there is a request sequence R such that for any $\varepsilon > 0$

$$\frac{\text{LRU}_k(R)}{\text{OPT}_k(R)} \geq k - \varepsilon,$$

where LRU_k and OPT_k denote the number of faults of LRU and OPT with cache size k , respectively.