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Sheet 06

Average-Case Analysis

Exercise 1 (To See or Not To See (a Movie))

Gretchen and Henry were sent to their rooms for fighting in the house. They each separately voiced their protest to their father, insisting that the fight was nothing more than healthy sibling competition, and they each wanted to go out that afternoon to see a movie. He was moved by their stories, but wouldn't simply set them free. Instead, he devised a system. He went to each child's room with a penny, and told them that they would have to show up in the den in 10 minutes, and could choose to bring the penny with them or leave it in their respective rooms. Dad would then flip the one or two pennies brought to the den, and if the pennies he flipped came up heads, the kids could go to the movies. If neither brought a penny, or if he flipped at least one tail, they would stay in their rooms until supper time.
The problem facing Gretchen and Henry was that neither knew what the other would do. It would be easy if they could collude – one would bring a penny, and the other would not, giving them a 50% chance of going free – but they did not have this luxury.
If they both acted optimally, what is the probability that they will be free in time to see the movie?

Exercise 2 (Bin Packing Lower Bound)

The task of this exercise is to show that there does *not* exist a polynomial time algorithm with approximation-guarantee *less than* 3/2 for (deterministic offline) Bin Packing, unless $\mathsf{P} = \mathsf{NP}$.
You may want to use that the problem Partition is NP-complete. For Partition, you are given items (having non-negative weights) with total weight B. The question is if it is possible to partition them in *two* sets with total weight $B/2$, each.
Hint. Proceed by contradiction assuming that there is such an algorithm mentioned for Bin Packing.

Exercise 3 (Two Item Types)

Suppose the weights W_i for $i = 1, \ldots, n$ are idependent and identically distributed such that $W_i = 1/3$ with probability 1/2 and $W_i = 2/3$ otherwise. For the stochastic Bin Packing compute

$$\mathbb{E}[L] \quad \text{and with that an upper bound for} \quad e'(\textsc{nf}) = \lim_{n\to\infty} \frac{\mathbb{E}[\textsc{nf}(W)]}{\mathbb{E}[\textsc{opt}(W)]}.$$

Hint. Notice that the L_j can have only two values, namely 2/3 and 1. For $k, \ell \in \{2/3, 1\}$ compute the $p_{k\ell} = \Pr[L_{j+1} = \ell \mid L_j = k]$. These give rise to a 2×2-matrix $P = (p_{k\ell})$. The limit distribution $\pi = (\pi_1, \pi_2)$ for L has to satisfy $\pi P = \pi$.