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Algorithms Theory, Assignment 3

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Exercise 3.1 - Universal hashing [Points: 5]

Let $U = \{0, \ldots, N-1\}$, where N is 49 and m is 35. Let $a_i = 42 \cdot i$ and $b_i = 28 \cdot i$. Now consider the following class of hash functions.

$$\mathcal{H} = \{h_i(k) = ((a_i \cdot k + b_i) \mod N) \mod m\} \text{ for } i \in \{1, \ldots, N(N-1)\}$$

Is $\mathcal{H}$ universal? Prove your answer.

Exercise 3.2 - Perfect hashing [Points: 3+2]

Let $U = \{0, \ldots, 28\}$ and $S = \{1, 3, 8, 9, 14, 18, 20, 21, 24, 27\}$.

1. Use the two-level scheme to build a perfect hash function with $k = 3, N = 29, n = 10$. For $i = 0, \ldots, n-1$ determine the values W_i, b_i, m_i, k_i, s_i, and h_{k_i}.
2. For each element of S provide the position in the hash table at which this element is stored.

Exercise 3.3 - Amortized Analysis [Points: 3+2]

1. For a data structure the i-th operation has costs c_i, where

   $$c_i = \begin{cases} i & \text{if } i \in \{2^k \mid k \in \mathbb{N}\} \\ 1 & \text{else} \end{cases}$$

   Prove by the aggregate method that the worst cases amortized cost of an operation is constant.

2. Suppose we have a potential function Φ such that $\Phi_0 \neq 0$ and $\Phi_i \geq \Phi_0$ for all i. Show that there exists a potential function Φ' such that $\Phi'_0 = 0$, $\Phi'_i \geq 0$, the total amortized costs resulting from Φ' are the same as the total amortized costs resulting from Φ.

Exercise 3.4 - Ternary counter [Points: 3+2]

Consider a Ternary counter, i.e. a counter of base 3 with digits 0,1 and 2. The counter starts at 0 and will be increased by 1 n-times. The cost of increasing the counter from $i-1$ to i is given by the number of digits that must be changed. Let $A(n)$ denote the cost of incrementing the counter from 0 to n. Then:

1. Give the minimal $c \in \mathbb{R}$ such that

   $$A(n) \leq cn \text{ for all } n \in \mathbb{N} \tag{1}$$

   holds. Use amortized analysis to show that c holds in equation 1.

2. Show that c is indeed minimal.
   Hint: Show that for every $\epsilon > 0$, $(c - \epsilon)$ in equation 1 does not hold.