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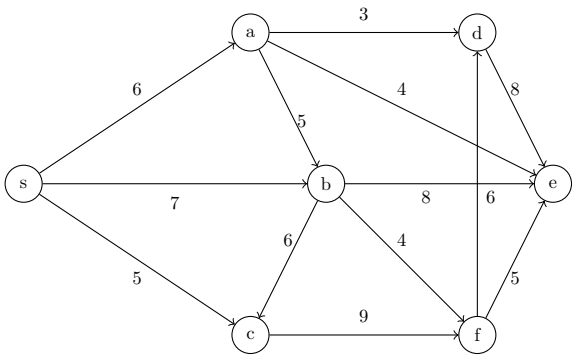
Algorithms Theory, Assignment 6

Submission: 26. Jan. 2011, 4 p.m.

Exercise 6.1 - Shortest path [Points: 5]

Consider the acyclic graph $G = (V, E, c)$ bellow in which all edges have non-negative weights. By using Dijkstra's algorithm compute the shortest paths from s to all other vertices. Give the DIST value of all vertices after every iteration and show the resulting rooted tree.



Exercise 6.2 - Dijkstra's algorithm [Points: 1+1+1+2]

Give the complexity of the Dijkstra's Algorithm if the min-priority queue is implemented by:

- List
- Binomial queue
- Min heap

Suppose we change line 3 of Dijkstra's algorithm to the following. 3 while $\mid U \mid > 1$. This change causes the while loop to execute $\mid U \mid -1$ times instead of $\mid U \mid$ times.

- Is this proposed algorithm correct?. Prove your answer.

Exercise 6.3 - Minimum spanning trees [Points: 5]

Consider a graph $G = (V, E)$ with weight function $c : E \to \mathbb{R}$. Now, assume that for every cut of G, there is a unique light edge (minimale Kante) crossing the cut. Show that G has a unique spanning tree T. In addition, by means of a counterexample, show that the converse is not true.

Exercise 6.4 - Bin packing [Points: 3+2]

Consider the following sequence I of items:

$$\underbrace{\frac{1}{43}+\epsilon,\ldots,\frac{1}{43}+\epsilon}_{42m},\underbrace{\frac{1}{7}+\epsilon,\ldots,\frac{1}{7}+\epsilon}_{42m},\underbrace{\frac{1}{3}+\epsilon,\ldots,\frac{1}{3}+\epsilon}_{42m},\underbrace{\frac{1}{2}+\epsilon,\ldots,\frac{1}{2}+\epsilon}_{42m} \quad (m \in \mathbb{N})$$

- Construct an optimal packing and the packing which results after applying the First Fit method. Provide $\mathsf{OPT}(I)$ and $\mathsf{FF}(I)$.
- Apply the offline strategy First Fit Decreasing to I. Construct the resulting packing and provide $\mathsf{FFD}(I)$.