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Sheet 02

Algorithm Theory

Exercise 1 (Polynomial Coefficients) [Points: 5]

Evaluating a polynomial $a(x)$ of degree-bound n at a given point x_0 can also be done by dividing $a(x)$ by the polynomial $(x - x_0)$ to obtain a quotient polynomial $q(x)$ of degree-bound $n-1$ and a remainder r, such that

$$a(x) \quad = \quad q(x) \cdot (x - x_0) + r$$

Clearly, $a(x_0) = r$. Show how to compute the remainder r and the coefficients of $q(x)$ in time $\Theta(n)$ from x_0 and the coefficients of a.

Exercise 2 (Interpolation) [Points: 5]

Interpolate the point-value representation

$$(1, 6), (i, 15 + 15i), (-1, -36), (-i, 15 - 15i)$$

with FFT to create the coefficient representation of the polynomial.

Exercise 3 (Fast Fourier Transform) [Points: 5]

Compute the product of the two polynomials

$$p(x) = 4 + 5x \text{ and } q(x) = 9 + 2x$$

using FFT and interpolation.

Exercise 4 (Fast Fourier Transform) [Points: 5]

Let A and B be two sets of integers in the range of $[0, m-1]$ where m is a power of two. Show that the following can be computed in $\mathcal{O}(m \cdot \log m)$ time with a single DFT:

1. All elements contained in the set $A + B = \{c \,|a \in A, b \in B, c = a + b\}$
2. For each $c \in [0, ..., 2m-2]$, the number $k_c = |\{\{a, b\} \in A \times B \,|a + b = c\}|$.

Hint: Find some polynomials p_A, p_B of degree less than m that represent the sets A and B.