Albert-Ludwigs-Universität Freiburg
Institut für Informatik
Dr. Alexander Souza

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Sheet 03

Combinatorial Optimization

Exercise 1 (Fractional Knapsack)

Let $c, w \in \mathbb{R}^n$ be non-negative vectors with $c_1/w_1 \geq c_2/w_2 \geq \cdots \geq c_n/w_n$. The Fractional Knapsack problem is the following LP:

$$\begin{aligned} \text{maximize} \quad & \sum_{j=1}^{n} c_j x_j, \\ \text{subject to} \quad & \sum_{j=1}^{n} w_j x_j \leq W, \\ & 0 \leq x_j \leq 1 \quad j = 1, \ldots, n. \end{aligned}$$

Let $k = \min\left\{ j \in \{1, \ldots, n\} : \sum_{i=1}^{j} w_i > W \right\}$. Show that an optimum solution for the Fractional Knapsack problem is given by the vector x with

$$\begin{aligned} x_j &= 1 && \text{for } j = 1, \ldots, k-1, \\ x_j &= \frac{W - \sum_{i=1}^{k-1} w_i}{w_k} && \text{for } j = k, \text{ and} \\ x_j &= 0 && \text{for } j = k+1, \ldots, n. \end{aligned}$$

Exercise 2 (Fractional Multi-Knapsack)

In the Multi-Knapsack problem, we are given m knapsacks, each having a capacity W_i for $i = 1, \ldots, m$, n items each having weight w_j for $j = 1, \ldots, n$, and costs c_{ij} when item i is packed into knapsack j. We may assume that $\sum_{i=1}^{m} W_i \geq \sum_{j=1}^{n} w_j$. The task is to pack *all* items into knapsacks such that all knapsack capacities are obeyed and the total cost is minimized.

Give an ILP for this problem. Relax it to an LP and give a combinatorial polynomial-time algorithm that solves the relaxation (without using linear programming).

Hint. Reduction to Minimum Cost Flow.

Programming 3 (Multi-Knapsack)

Give OPL models for both, the Fractional Multi-Knapsack and the Multi-Knapsack problem with ILOG CPLEX. Notice that the latter is an ILP.