Albert-Ludwigs-Universität Freiburg
Institut für Informatik
Dr. Alexander Souza

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Sheet 04

Combinatorial Optimization

Exercise 1 (Greedy Algorithm for Knapsack)

Give an instance which shows that the approximation guarantee of 1/2 for the GREEDY algorithm for KNAPSACK is tight.

Exercise 2 (Hardness of Knapsack)

Show that the decision-version of KNAPSACK is NP-complete. This refers to the problem: For given $c, w \in \mathbb{N}^n$ and $W, C \in \mathbb{N}$ is there a solution $x \in \{0,1\}^n$ with $\text{weight}(x) = \sum_{j=1}^n w_j x_j \leq W$ and $\text{val}(x) = \sum_{j=1}^n c_j x_j \geq C$?
Hint. Reduction from PARTITION: For given $a_1, \ldots, a_n \in \mathbb{N}$ is there a subset $S \subseteq \{1, \ldots, n\}$ such that $\sum_{i \in S} a_i = \sum_{i \notin S} a_i$? PARTITION is known to be NP-complete.

Exercise 3 (Vertex Cover)

The problem VERTEX COVER is defined as follows: Given a simple undirected Graph $G = (V, E)$ find a set $C \subseteq V$ with minimal cardinality, such that each edge in E has at least one endvertex in C.
Give a 2-approximation for VERTEX COVER.
Hint. Consider the edges of the graph. For some of the edges, include both endvertices.