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WS 2011/12
Sheet 05

Combinatorial Optimization

Exercise 1 (Set Cover Greedy)

Give an instance showing that the GREEDY algorithm for SET COVER is a H_n-approximation.

Exercise 2 (Weighted Vertex Cover)

The problem WEIGHED VERTEX COVER is defined as follows: Given a simple undirected Graph $G = (V, E)$ and a cost function on the vertices $c : V \to \mathbb{R}^+$, find a set $C \subseteq V$ with minimal cost, such that each edge in E has at least one endvertex in C. The cost of $C \subseteq V$ is $\text{val}(C) = \sum_{i \in C} c(i)$.
Give a 2-approximation for WEIGHTED VERTEX COVER.
Hint. Formulate the problem as a SET COVER problem.

Exercise 3 (Maximum Coverage)

The MAXIMUM COVERAGE problem is the following: Given a universe $U = \{u_1, \ldots, u_n\}$ of n elements, with non-negative weights $w : U \to \mathbb{R}^+$, a collection of subsets $\mathcal{S}$ of U, and an integer k, pick k sets so as to maximize the weight of covered elements.
Consider the following GREEDY algorithm:

Step 1. Set $G_0 = \emptyset$.

Step 2. For $i = 1, \ldots, k$:

- (a) Select S that maximizes $\sum_{u \in G_{i-1} \cup S} w(u)$.
- (b) Set $G_i = G_{i-1} \cup S$.

Step 3. Return G_k.

Show that this algorithm achieves an approximation guarantee of $1 - (1 - 1/k)^k$. Notice that this is at least $1 - 1/e \approx 0.632$.
Hint. Firstly, show that the weight added in each iteration is at least a $1/k$ fraction of the weight difference to the optimal solution G^*, i.e., $\text{weight}(G_i) - \text{weight}(G_{i-1}) \geq 1/k \cdot (\text{weight}(G^*) - \text{weight}(G_{i-1}))$. Secondly, prove $\text{weight}(G_i) \geq (1 - (1 - 1/k)^i) \cdot \text{weight}(G^*)$.