New Classes of Distributed Time Complexity

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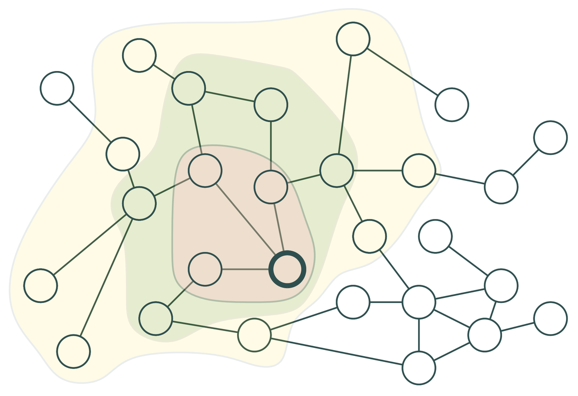
Context and Goals

- Study locally checkable labelling (LCL) problems in the LOCAL model
- Understanding the complexity landscape of LCL problems on general graphs

The LOCAL Model

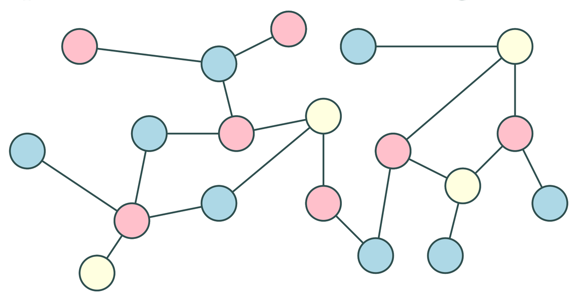
- Synchronous model
- Nodes have IDs
- No limits on bandwidth or computational power



Locally Checkable Labellings

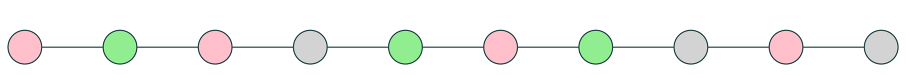
- Introduced by Naor and Stockmeyer in 1995 [2]
- Δ–bounded degree graphs (where Δ is a constant)
- Constant-size input and output labels
- Validity of the output is locally checkable

Example: Vertex Colouring

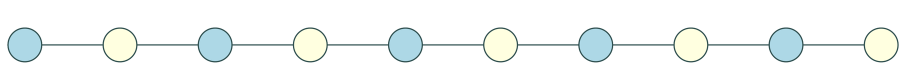


LCLs on Cycles and Paths

- $\Theta(1)$: trivial problems
- $\Theta(\log^* n)$: local problems (symmetry breaking)



- $\Theta(n)$: global problems



Landscape of Complexities on Cycles and Paths

LCLs on Trees

- Any $n^{o(1)}$ rounds algorithm can be converted to an $O(\log n)$ rounds algorithm [3]
- There are problems of complexity $\Theta(n^{1/k})$ [3]

Landscape of Complexities on Trees

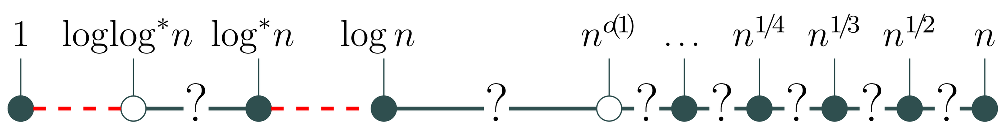
Conjecture on Trees

Towards Proving the Conjecture on Trees [4]

LCLs on General Graphs

- There are problems with complexity $\Theta(\log n)$
- Any $o(\log \log^* n)$ rounds algorithm can be converted to an $O(1)$ rounds algorithm (same techniques of [2])
- Any $o(\log n)$ rounds algorithm can be converted to an $O(\log^* n)$ rounds algorithm [5]
- Many problems require $\Omega(\log n)$ and $O(\text{poly} \log n)$ rounds

Landscape of Complexities on General Graphs



Conjectures

A Motivating Example

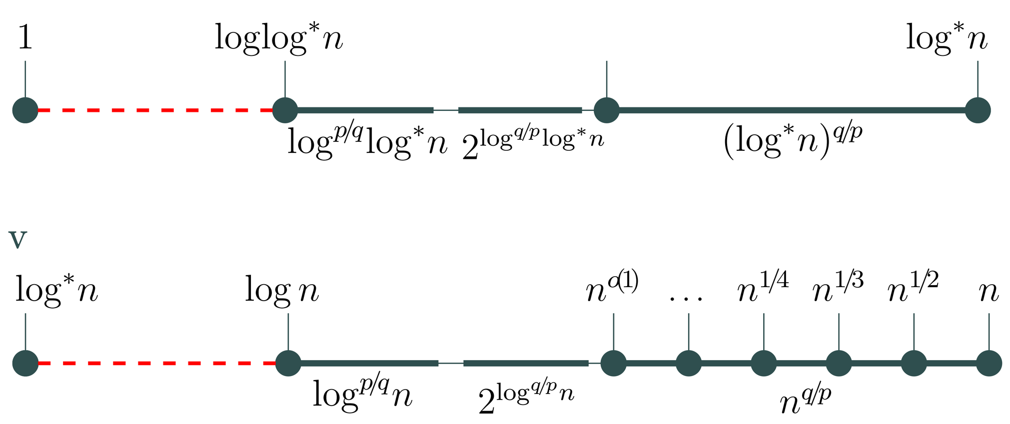
- Δ–colouring in general graphs can be done in $O(\text{polylog}\, n)$ rounds
- 4–colouring a 2–dimensional balanced grid can be done in $O(\text{polylog}\, n)$ rounds
- In 2–dimensional grids, there is a gap between $\omega(\log^* n)$ and $o(\sqrt{n})$ [6]
- *Implication*: 4–colouring a 2–dimensional balanced grid can be done in $O(\log^* n)$ rounds

Our Results

Complexities on General Graphs [1]

Latest (Unpublished) News [4]

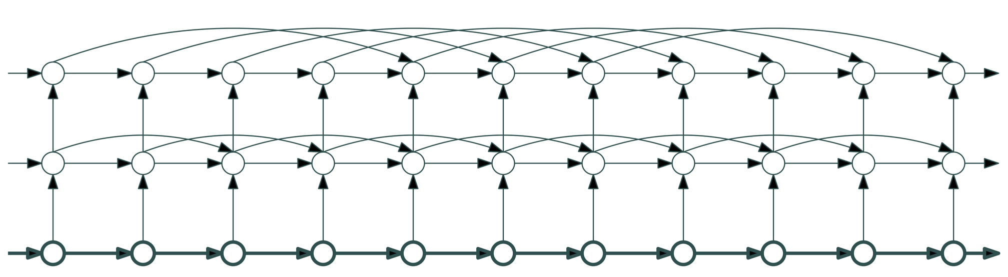
Low vs High Complexities



Proof Ideas

- Start from an LCL problem Π on cycles

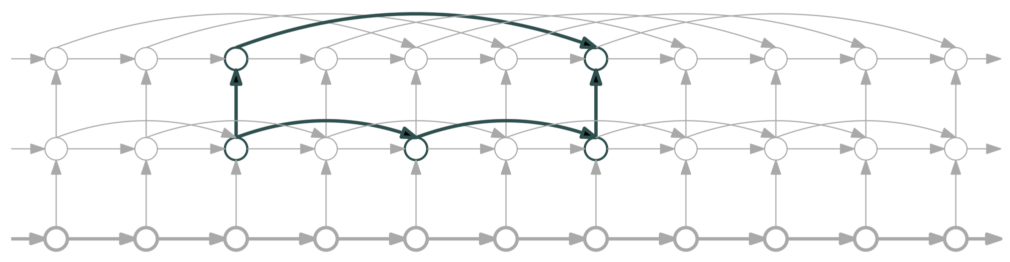
- Build a speed-up construction
- Example: exponential speed-up function (2^{ℓ}, where ℓ is the level of the grid-like structure)



A Valid LCL

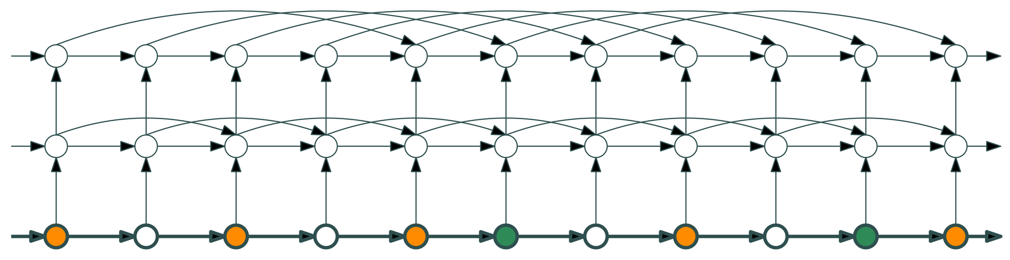
An LCL problem must be defined on any graph, not just on some "relevant" instances

Local Checkability of the Input Graph

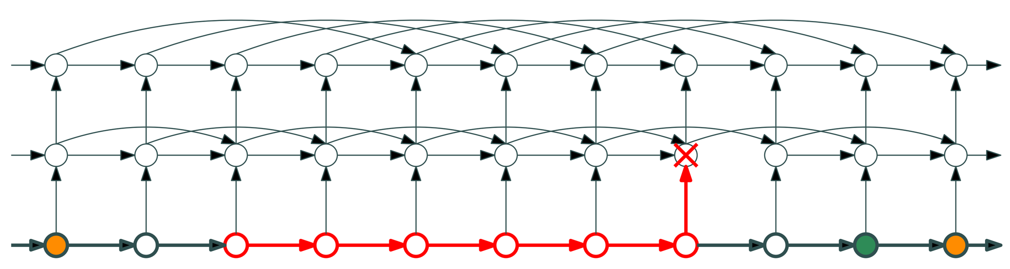


On Correct Instances

- $T(n) = \Theta(\log^* n)$ for 3–vertex colouring on cycles
- $T(n) = \Theta(n)$ for 2–vertex colouring on cycles
- Problem Π can be solved in $o(T(n))$ rounds using the shortcuts

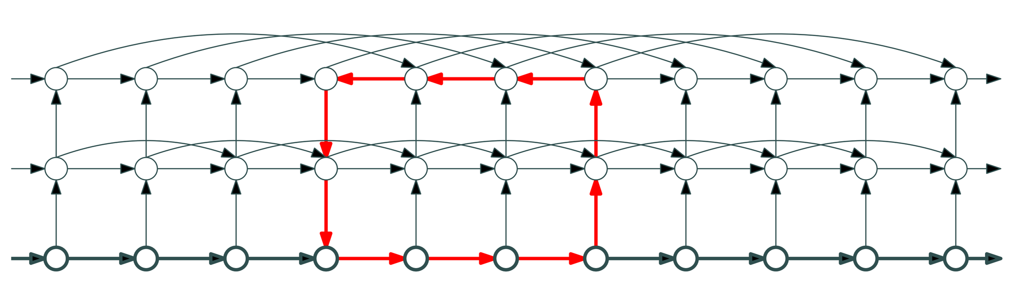


On Incorrect Instances



Hardness Balance

- On incorrect instances, it should be easy to prove that there is an error
- On correct instances, it should be impossible, or hard, to prove that there is an error



Open Problems

- What happens between $\Omega(\log \log^* n)$ and $O(\log^* n)$ on trees?
- What are meaningful subclasses of LCL problems worth studying?

References

[1] A. Balliu, J. Hirvonen, J. H. Korhonen, T. Lempiäinen, D. Olivetti, and J. Suomela, "New classes of distributed time complexity," in *STOC 2018 (to appear)*.

[2] M. Naor and L. Stockmeyer, "What can be computed locally?," *SIAM Journal on Computing*, 1995.

[3] Y. Chang and S. Pettie, "A time hierarchy theorem for the LOCAL model," in *FOCS 2017*.

[4] A. Balliu, S. Brandt, D. Olivetti, and J. Suomela, "Almost global problems in the LOCAL model," 2018 (unpublished). https://arxiv.org/abs/1805.04776.

[5] Y. Chang, T. Kopelowitz, and S. Pettie, "An exponential separation between randomized and deterministic complexity in the LOCAL model," in *FOCS 2016*.

[6] S. Brandt, J. Hirvonen, J. H. Korhonen, T. Lempiäinen, P. R. Östergård, C. Purcell, J. Rybicki, J. Suomela, and P. Uznański, "LCL problems on grids," in *PODC 2017*.