New Classes of Distributed Time Complexity

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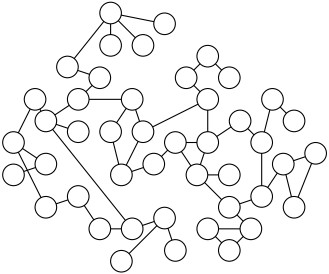
LOCAL Model



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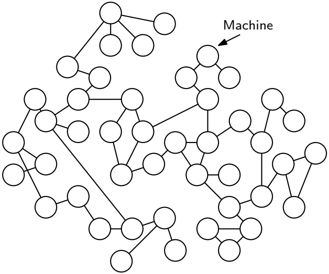

- Distributed network
- Nodes represent machines

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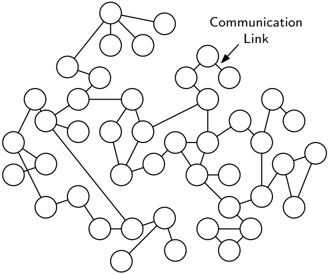

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- Nodes represent machines
- Edges represent communication links

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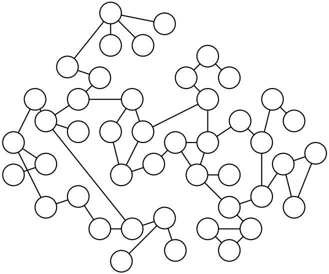

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- Synchronous

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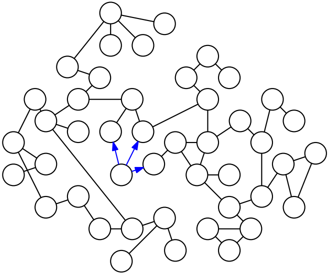

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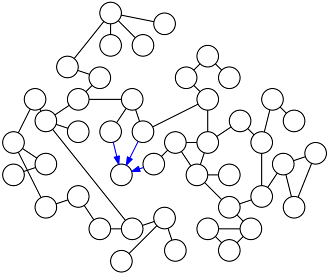

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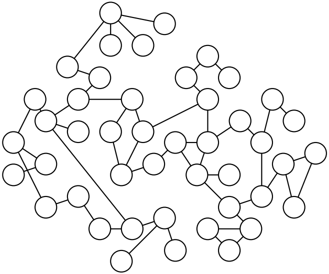

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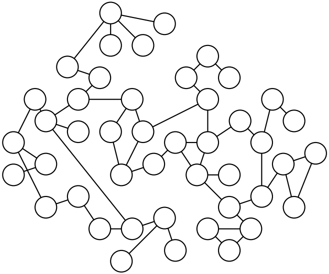

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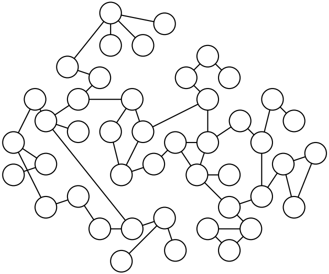

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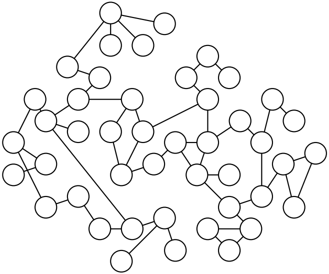

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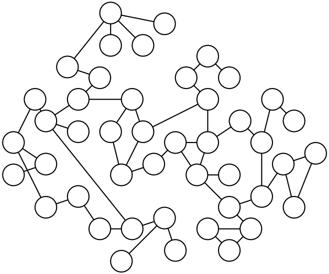

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- Messages of arbitrary size, arbitrary computational power
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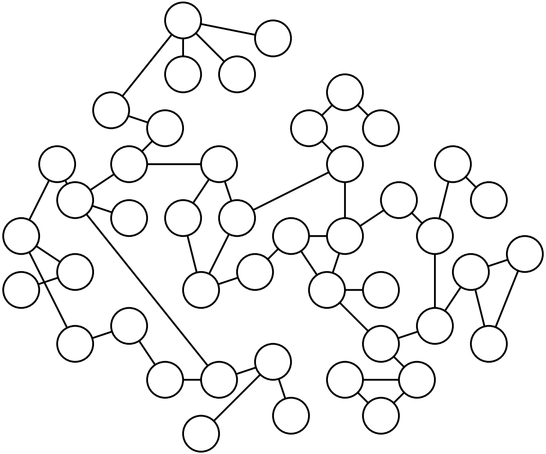
- Nodes know the size of the graph
- Complexity measure: number of rounds required to solve a task

LOCAL Model: easier description



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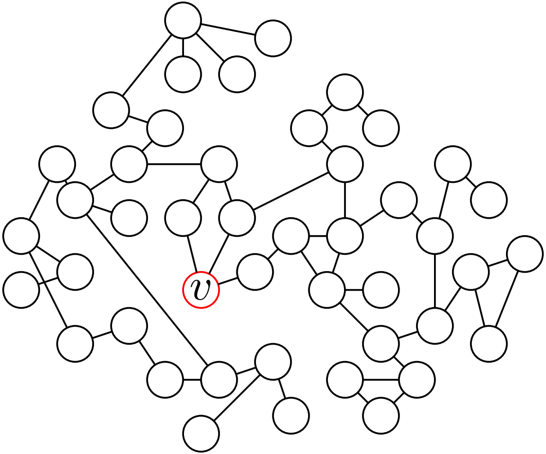

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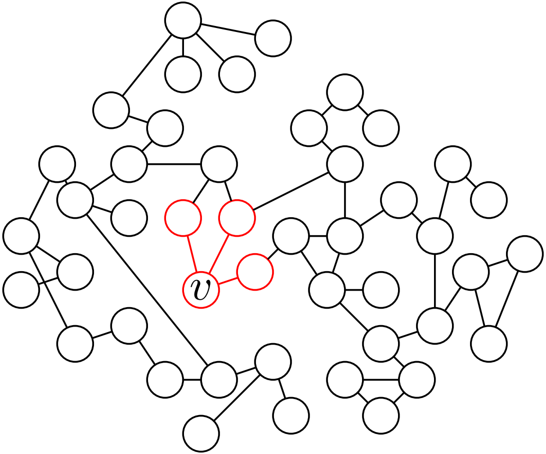

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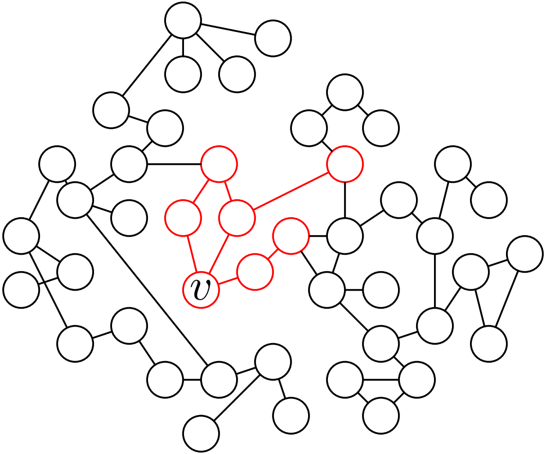

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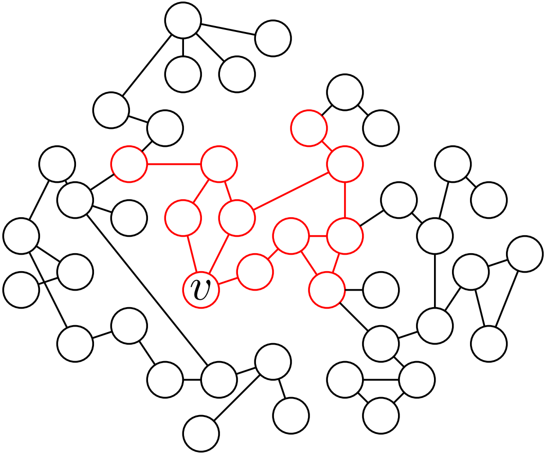

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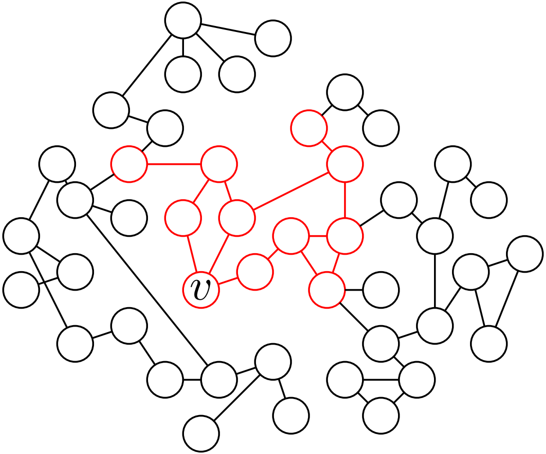


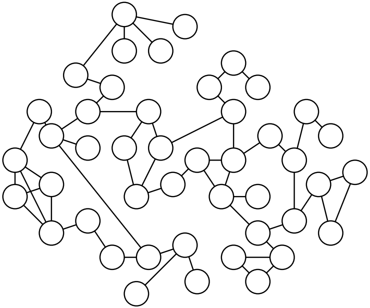
A t-round algorithm for the LOCAL model is a mapping from t-radius balls to valid outputs.

Locally Checkable Labellings

LCL Problems:

- Introduced by Naor and Stockmeyer in 1995
- Constant-size input labels
- Constant-size output labels
- The maximum degree is constant
- Validity of the output is locally checkable

Locally Checkable Labellings (Example)



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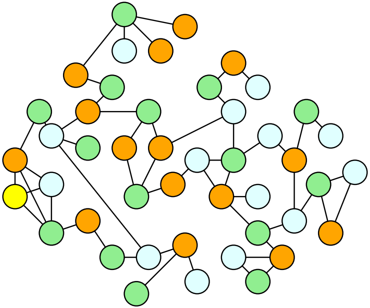

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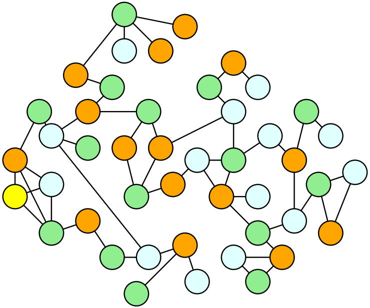

$\Delta + 1$ vertex colouring:

- The input is empty
- The output is in $\{1, \ldots, \Delta + 1\}$
- Nodes can check in 1 round if the colouring is valid

Local checkability

There must be a constant time distributed algorithm that is able to check the solution, such that:

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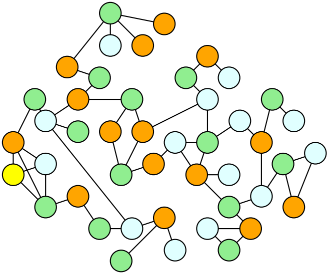

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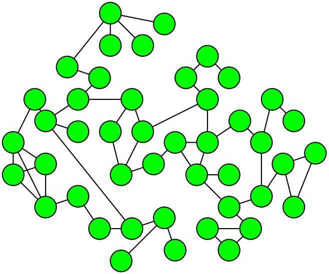

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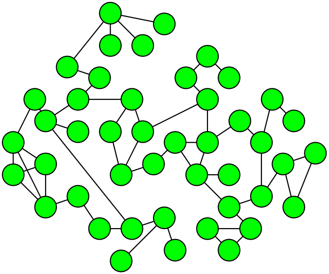

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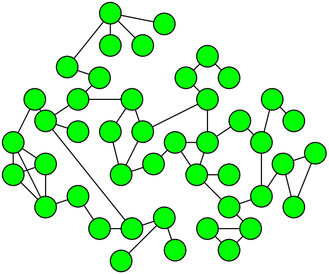

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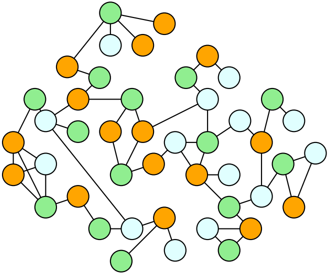

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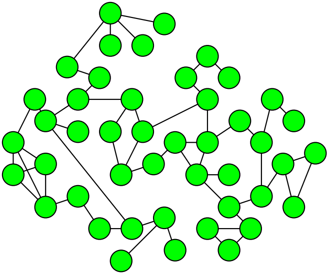

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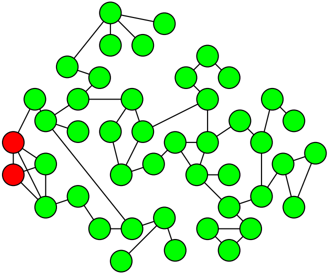


Locally Checkable Labellings (Motivation)

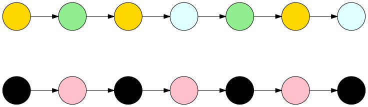
- Study the complexity of problems where the solution can be checked efficiently (like NP!)
- By restricting to constant degree graphs, we study problems related to distance, while ignoring the influence of other factors.
- It is a simple class that contains many well known problems.
- Lower bounds in this model apply to less powerful models.

Question

What are the possible time complexities for LCL problems?

LCL on Cycles and Paths

- There are only three possible time complexities:
  - $\Theta(1)$: trivial problems
  - $\Theta(\log^* n)$: local problems (symmetry breaking)
  - $\Theta(n)$: global problems



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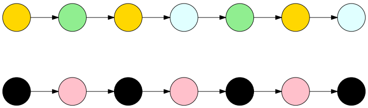

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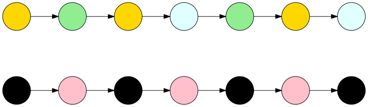
- On cycles with no input, given an LCL description, we can *decide* its time complexity. [Naor and Stockmeyer, 1995] [Brandt et al, 2017]



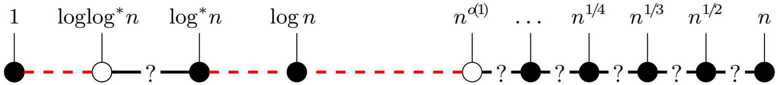
LCL on Cycles and Paths

LCL on Trees

[Chang and Pettie, 2017]:

- Any $n^{o(1)}$-rounds algorithm can be converted to a $O(\log n)$-rounds algorithm
- There are problems of complexity $\Theta(n^{1/k})$

LCL on Trees



LCL on General Graphs

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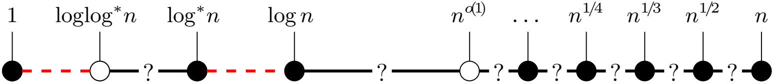
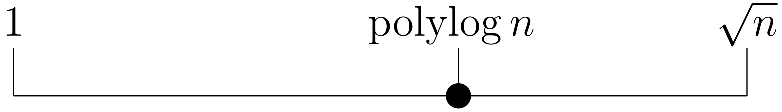
- Many problems require $\Omega(\log n)$ and $O(\text{poly} \log n)$
- Different scenario with randomized algorithms

LCL on General Graphs



Motivating Example

- Δ–colouring in general graphs can be done in $O(\text{polylog } n)$ rounds [Panconesi, Srinivasan 1995]
- 4–colouring in 2–dimensional balanced grids can be done in $O(\text{polylog } n)$ rounds



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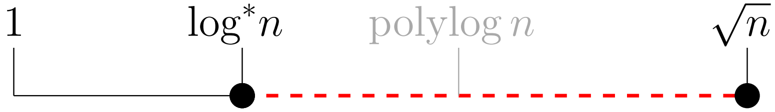

[Brandt et al. 2017]

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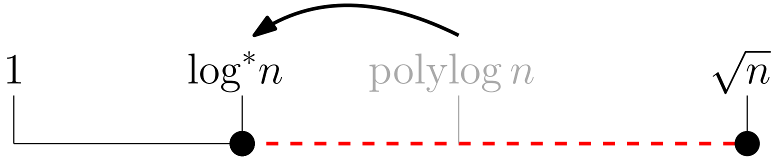
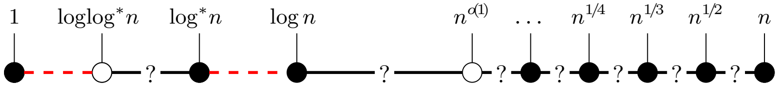


LCL on General Graphs?



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LCL on General Graphs (Our Results)

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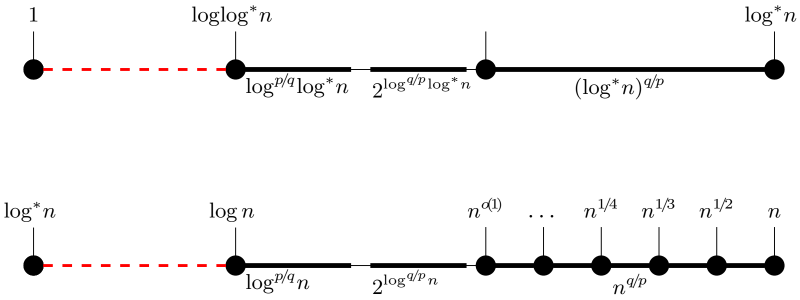

Proof idea

Counter Machine

- Registers
  $r_1, \ldots, r_k$
- Reset
  $r_a = 1$
- Addition
  $r_a = r_b + r_c$
  $r_a = r_b + \text{constant}$
- if $r_a = r_b$

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Counter Machine $\Rightarrow$ Distributed Complexity

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$T = f(g(t))$

Conclusions and Open Problems

- What happens between $\Omega(\log \log^* n)$ and $O(\log^* n)$ on trees?
- Can we prove automatic speedups for some subclass of LCL problems?

Thank you!

Questions?