Hardness of Minimal Symmetry Breaking in Distributed Computing

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Aalto University

An Automatic Speedup Theorem for Distributed Problems

Sebastian Brandt
ETH Zurich

Minimal Symmetry Breaking

Automatic Speedup Theorem

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Automatic Speedup Theorem

Tight Lower Bound for Weak 2-Coloring

Minimal
Symmetry
Breaking

Automatic
Speedup
Theorem

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Tight Lower Bound for Weak 2-Coloring

even-degree graphs

Tight Lower Bound for Weak 2-Coloring

odd-degree graphs

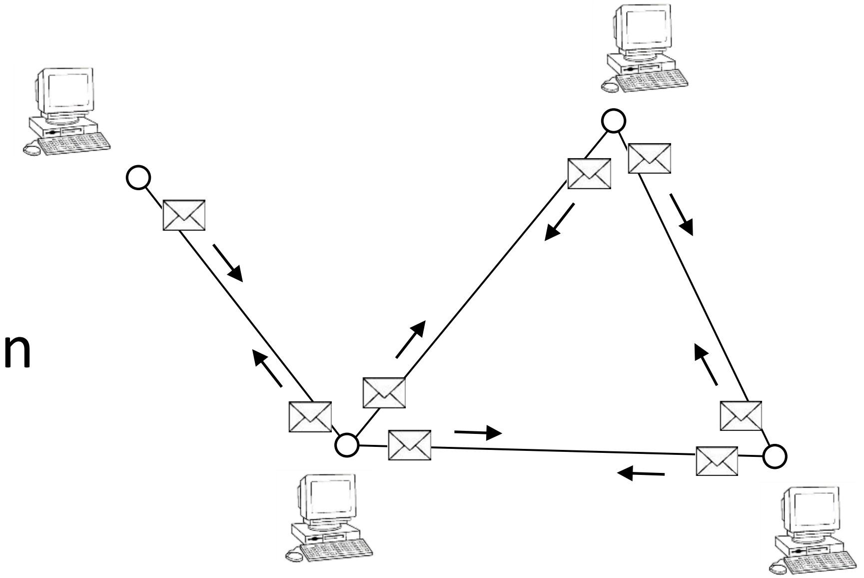
The LOCAL Model

- Synchronous rounds of
  1) Communication
  2) Computation

- Unlimited Message Size and Computation

- Runtime = number of rounds

- $O(\log n)$-bit unique identifiers



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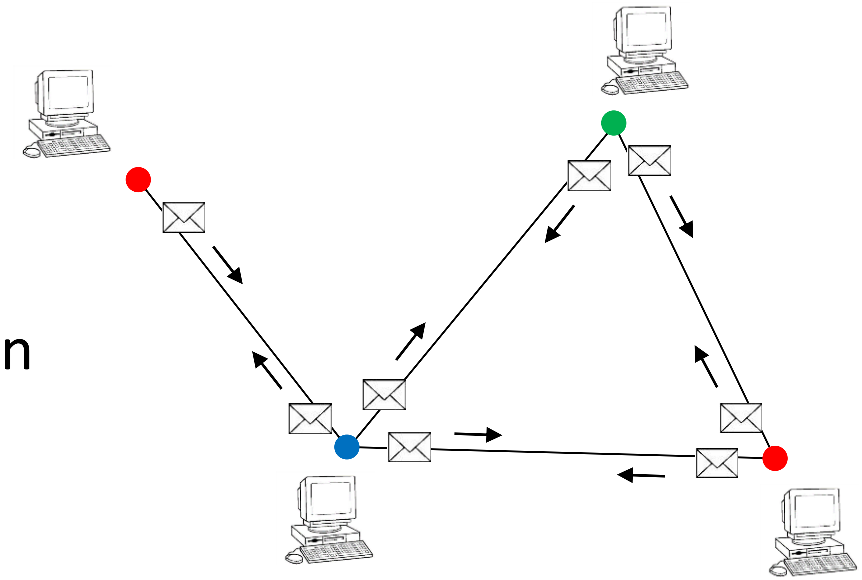

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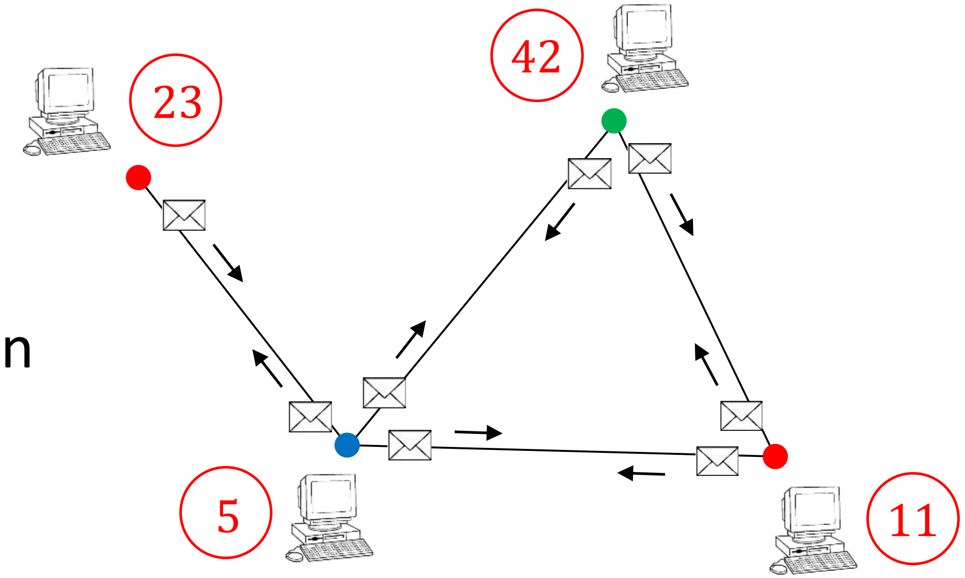
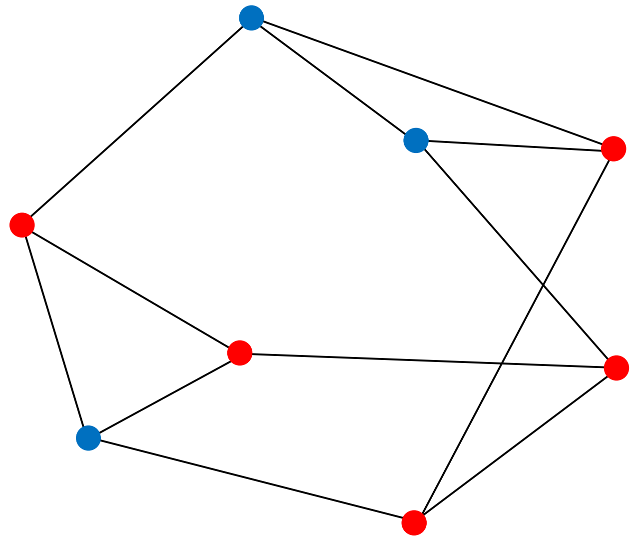


Weak k-Coloring

<u>Weak k-Coloring Problem:</u>

- k node colors
- each node has at least one neighbor of a different color



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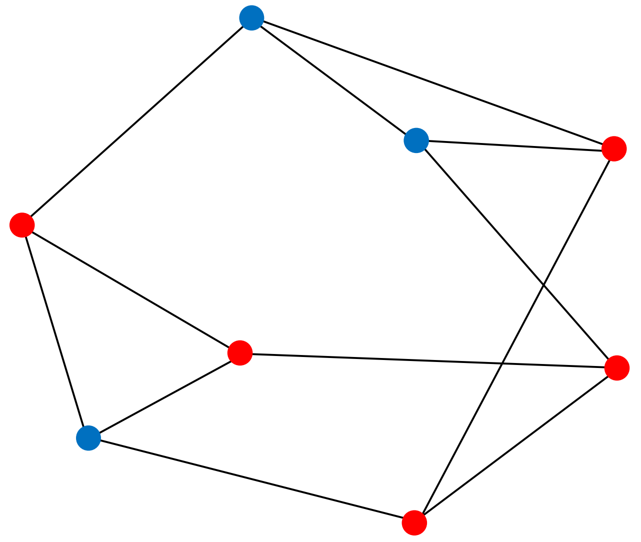
[Naor, Stockmeyer, STOC'93]

even-degree case

odd-degree case



Minimal Symmetry Breaking

Automatic Speedup Theorem

Even-Degree Weak 2-Coloring requires $\Omega(\log^* n)$ rounds.

Odd-Degree Weak 2-Coloring requires $\Omega(\log^* \Delta)$ rounds.

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Why study Weak 2-Coloring?

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Even-Degree Weak 2-Coloring is LOGSTAR-minimal.

class of symmetry-breaking problems

easiest problem in some class

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New general distributed lower bound technique

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↓ minimality

new lower bounds

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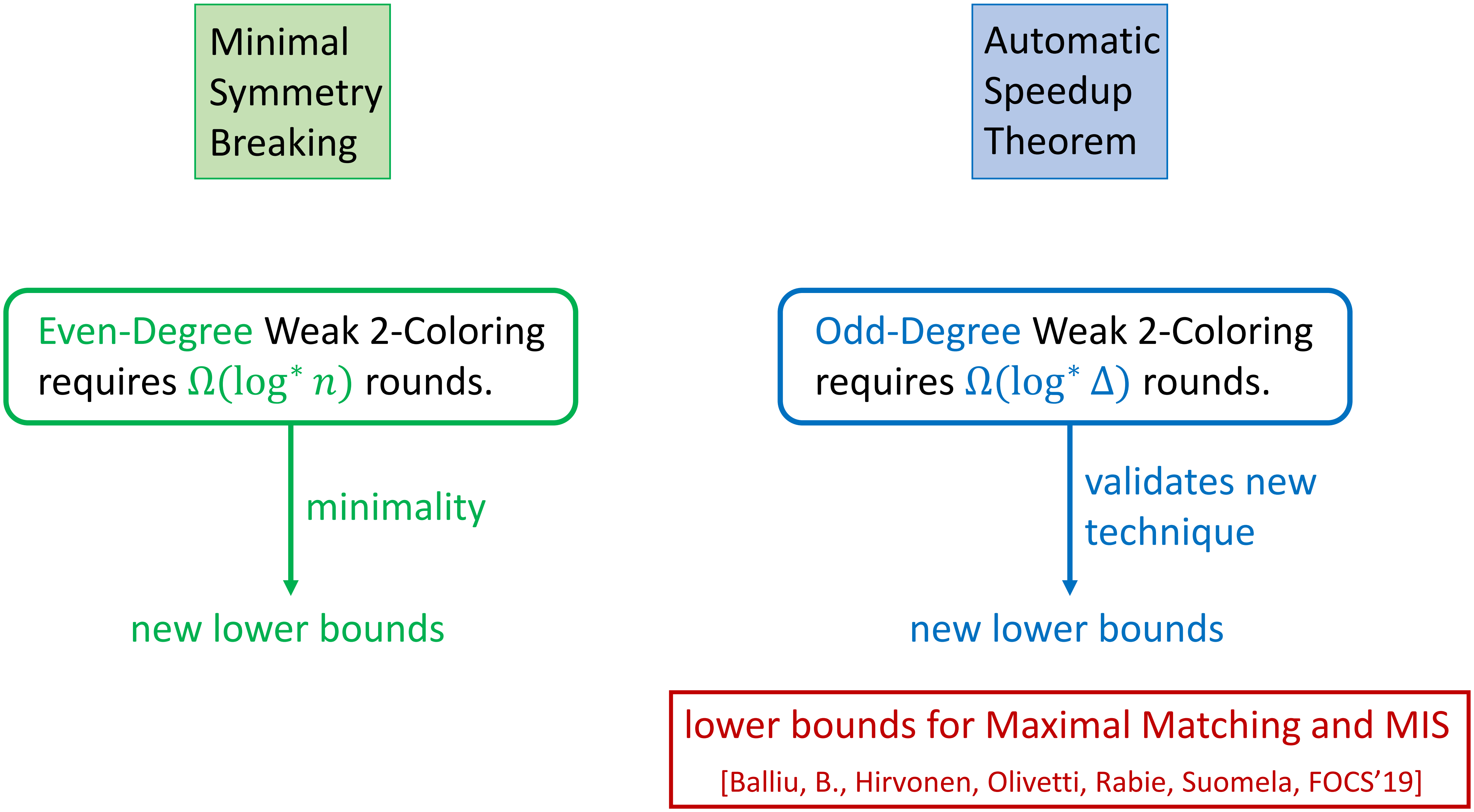

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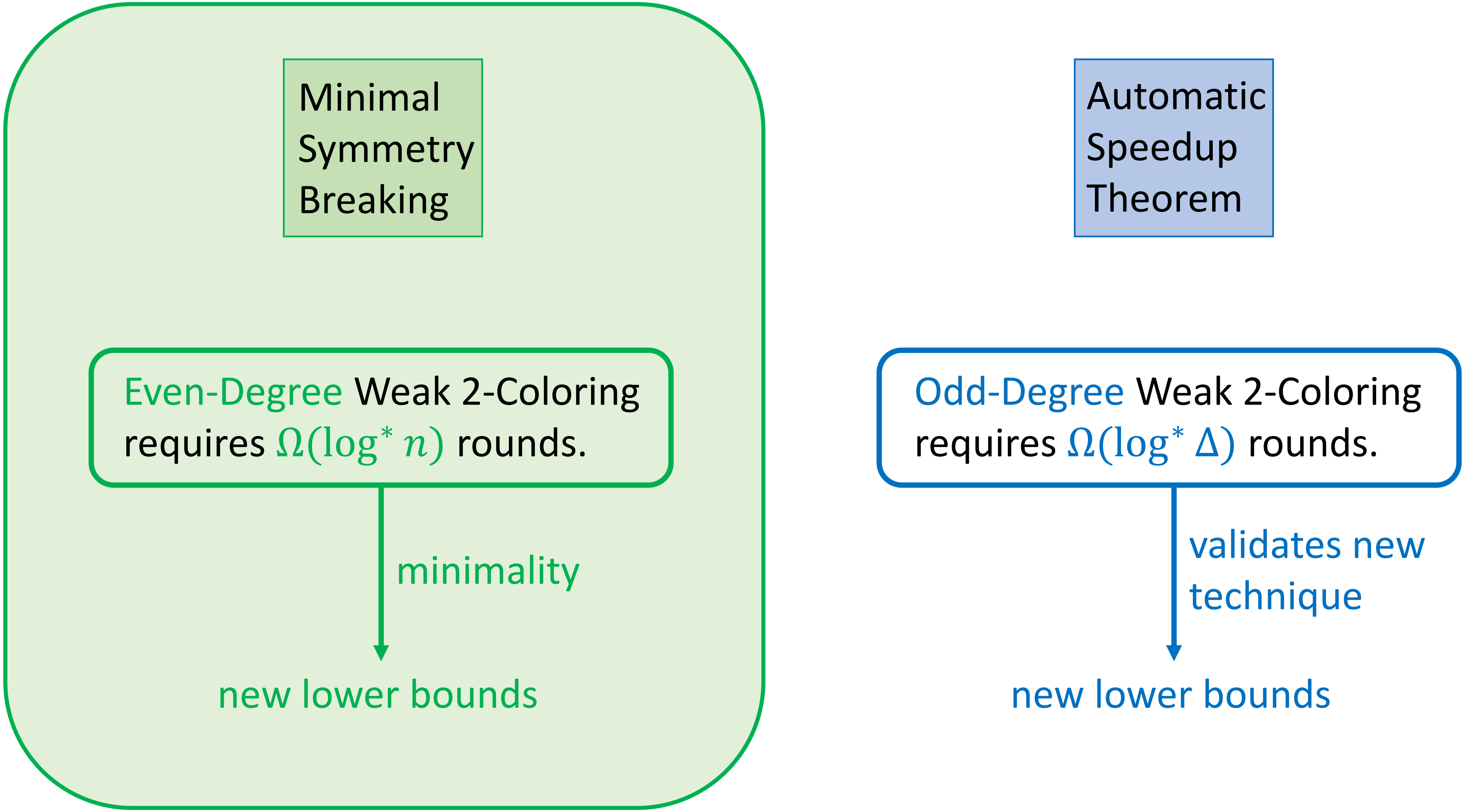
new lower bounds
lower bounds for Maximal Matching and MIS
[Balliu, B., Hirvonen, Olivetti, Rabie, Suomela, FOCS'19]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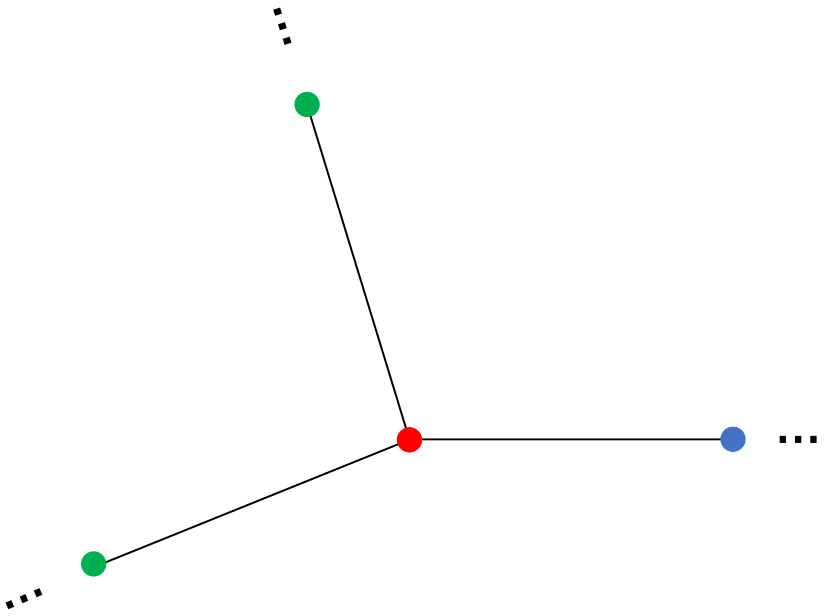
Locally Checkable Problems



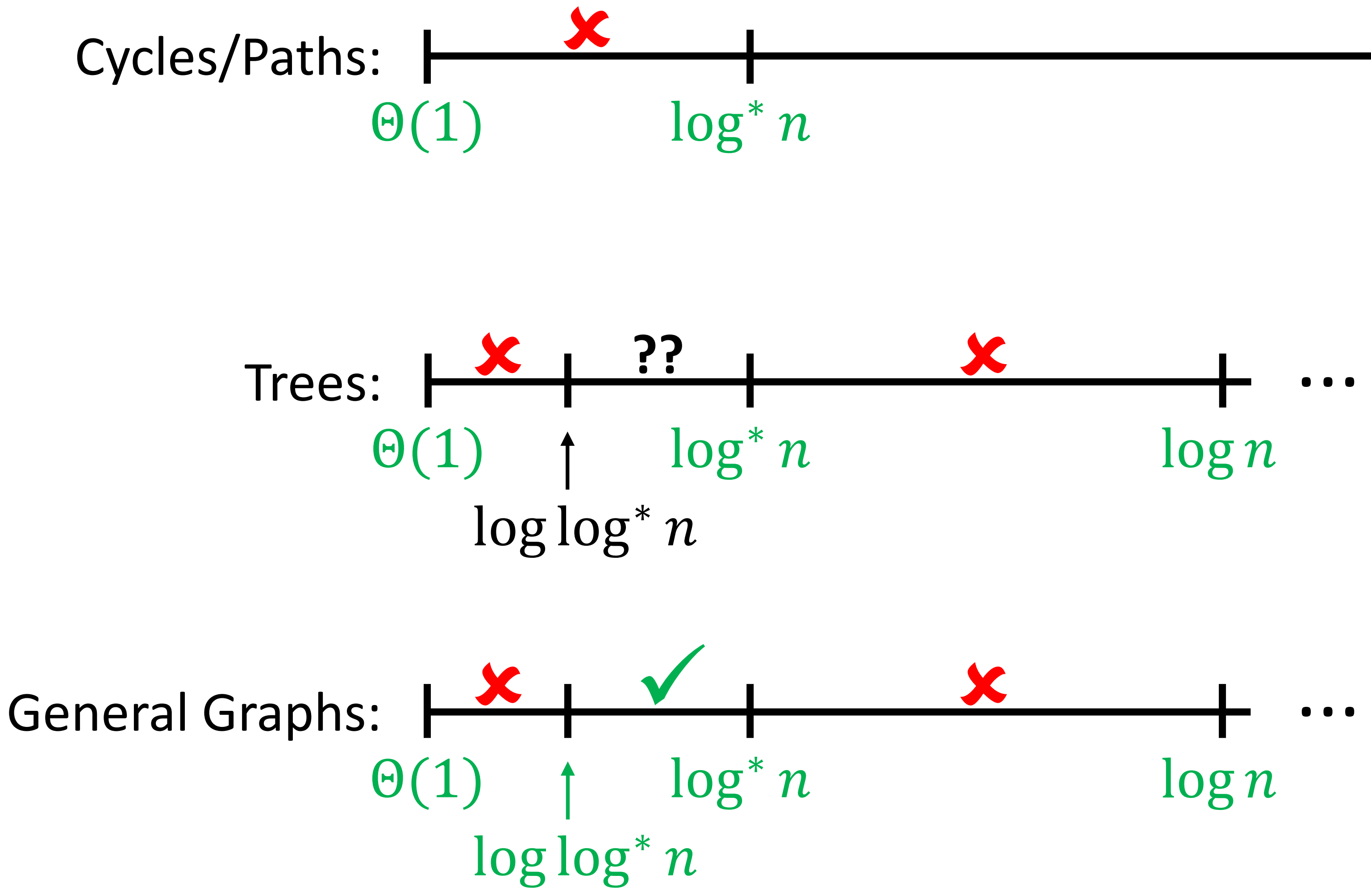
Locally Checkable:

Output correctness is defined via local ($= O(1)$-hop) constraints.

LCL Problems:

- bounded degree
- constant number of constraints

Complexity Landscape of LCL Problems



[Naor, Stockmeyer, STOC’93]
[B., Fischer, Hirvonen, Keller, Lempiäinen, Rybicki, Suomela, Uitto, STOC’16]
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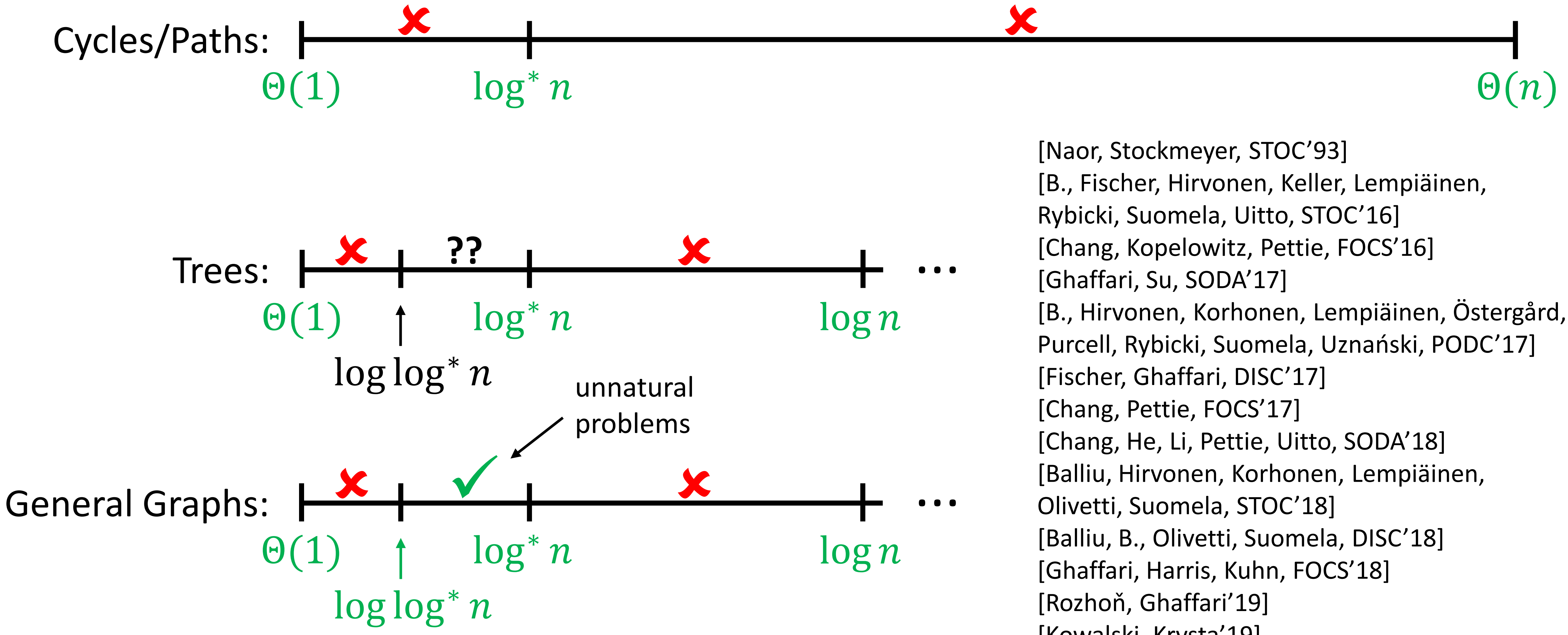

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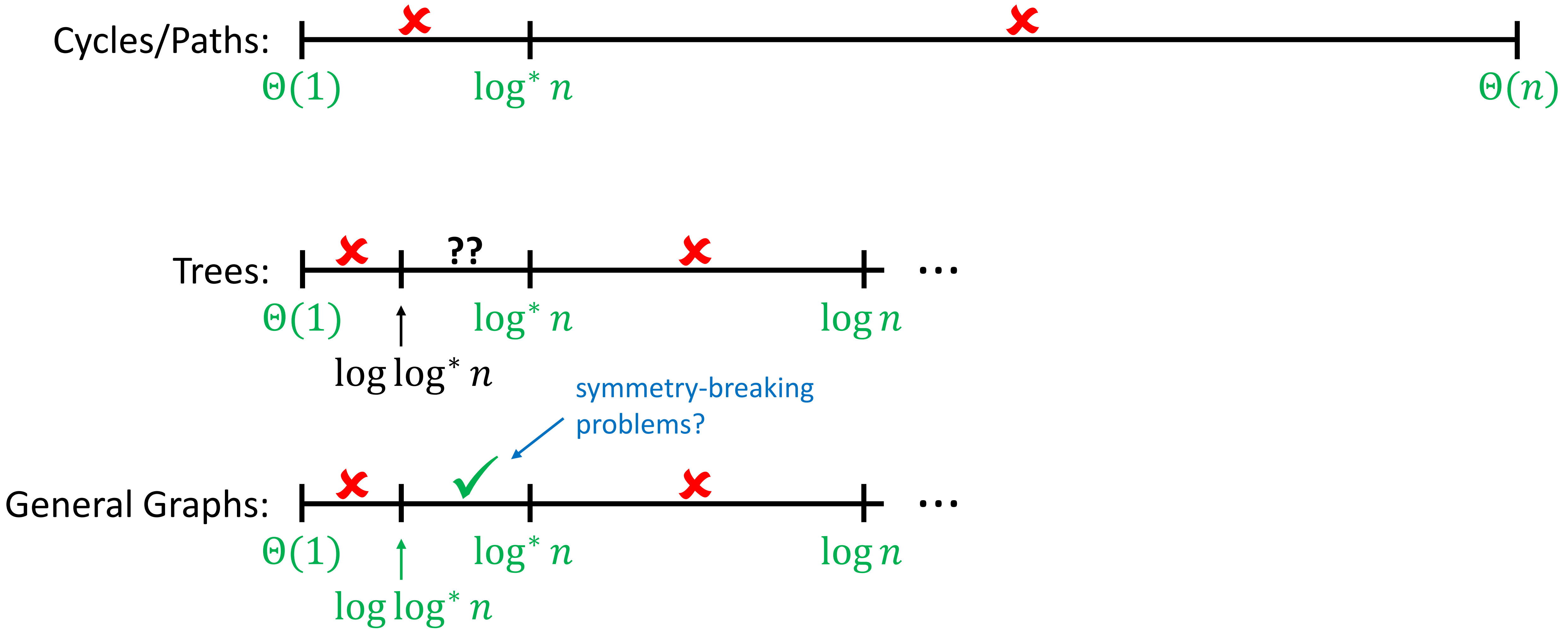

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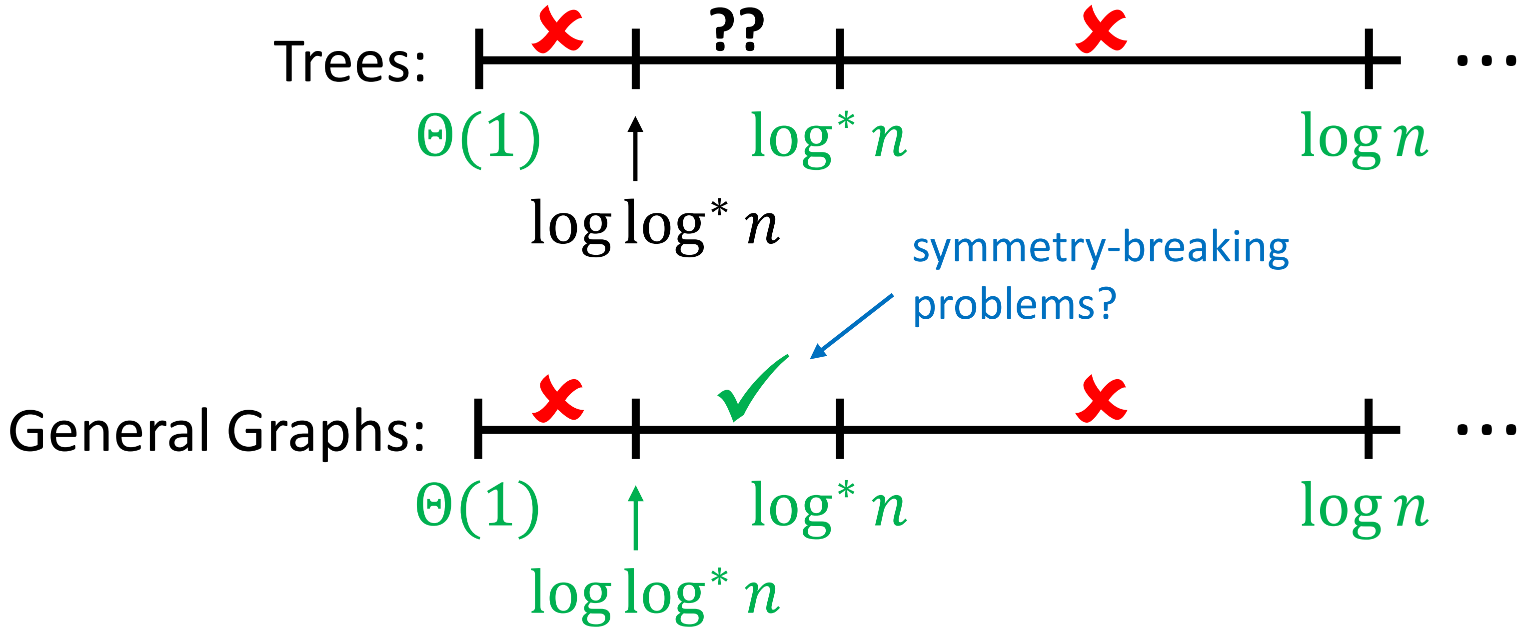

Homogeneous LCL:

Node sees an irregularity

↓

exempt from solving the LCL

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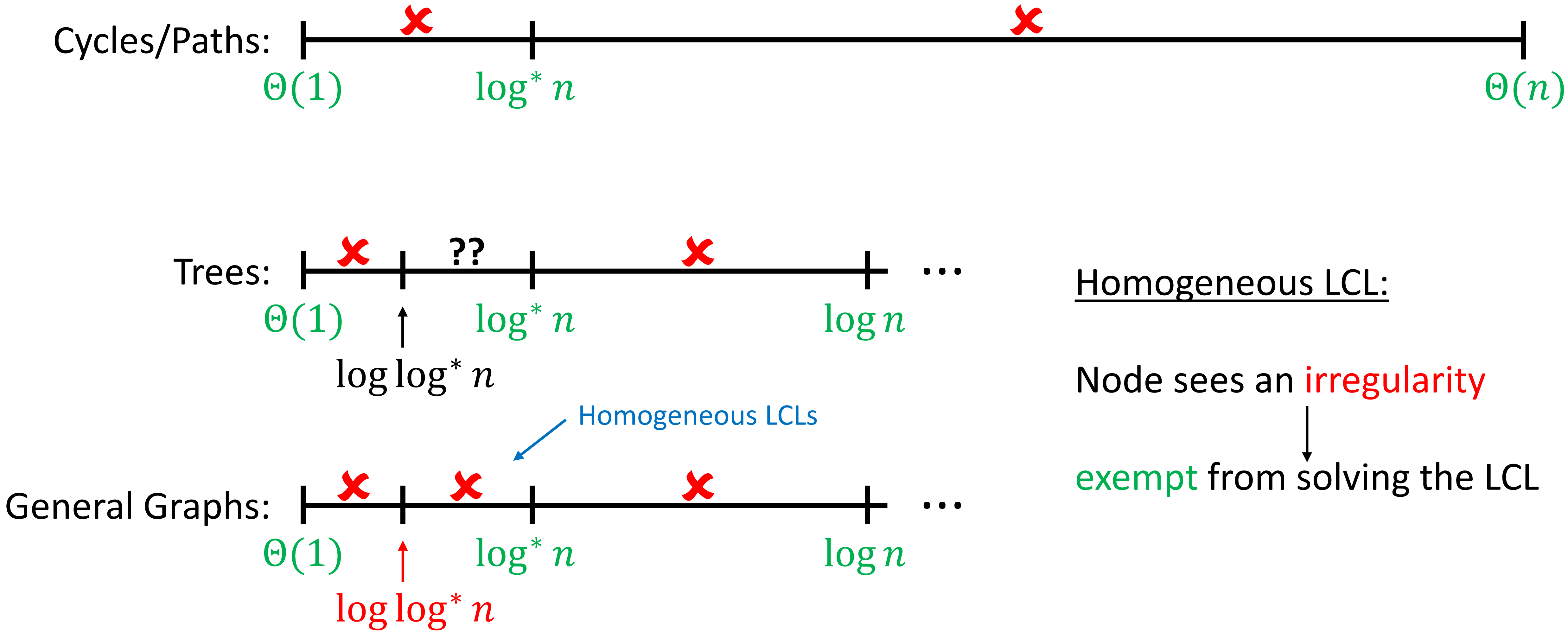

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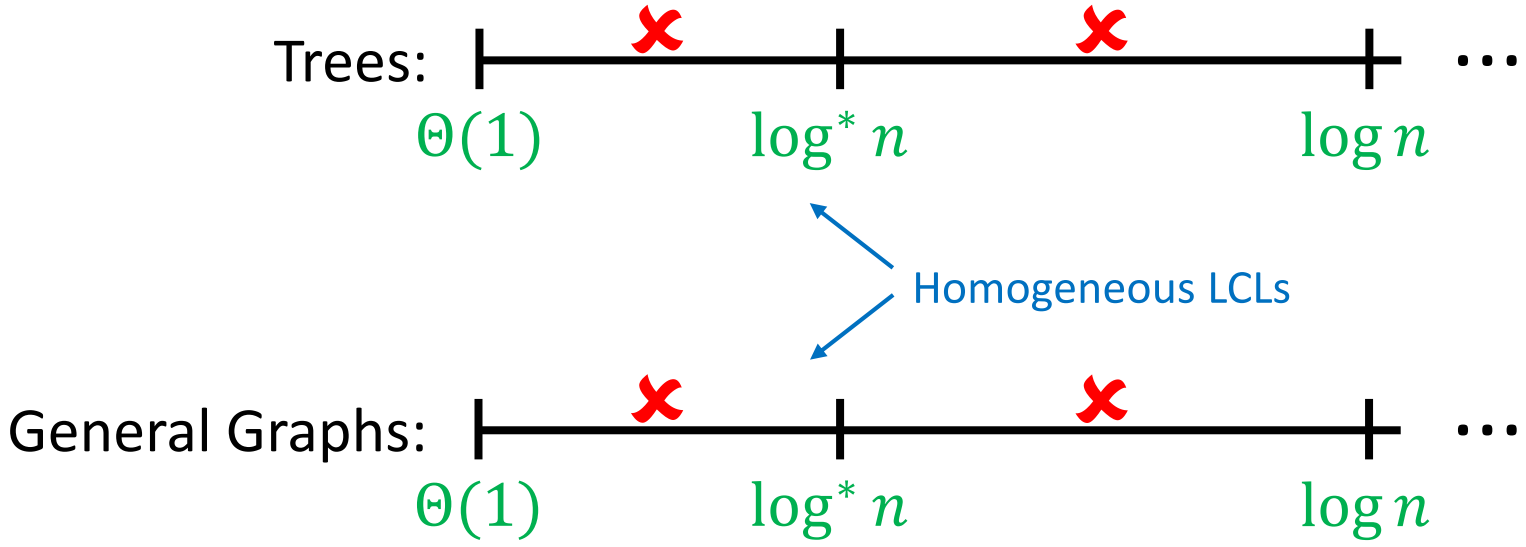

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Complexity Classification of Homogeneous LCLs

Deterministic	Randomized	
$\Theta(\log n)$	$\Theta(\log n)$	2-coloring
$\Theta(\log n)$	$\Theta(\log \log n)$	sinkless orientation
$\Theta(\log^* n)$	$\Theta(\log^* n)$	weak 2-coloring (even-degree graphs)
$\Theta(1)$	$\Theta(1)$	weak 2-coloring (odd-degree graphs)

Proof of the $\omega(1) - o(\log^* n)$ gap

LOGSTAR: class of homogeneous LCLs between $\omega(1)$ and $O(\log^* n)$

Even-Degree Weak 2-Coloring

↓ constant-time reduction

any $\mathcal{P} \in$ LOGSTAR

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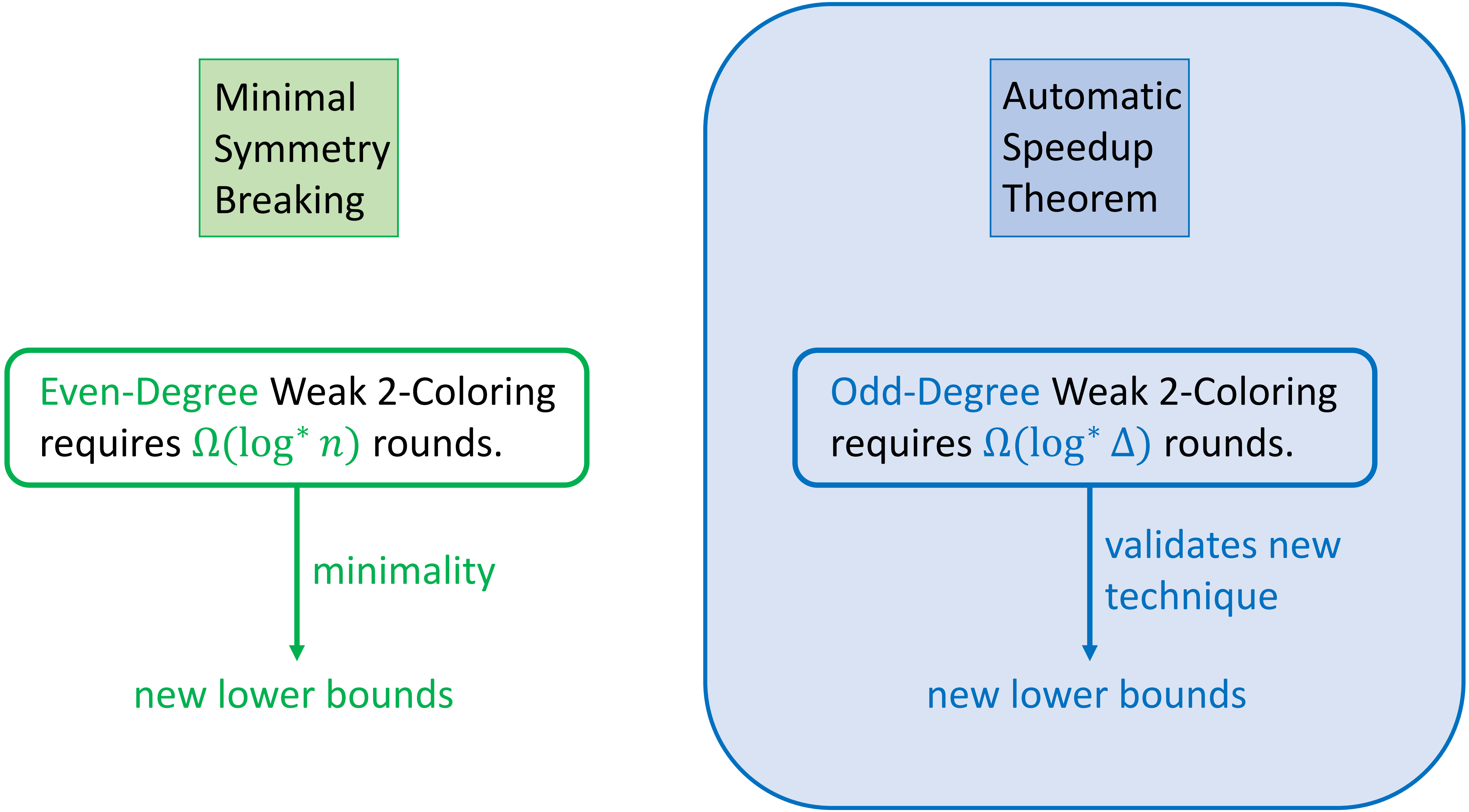
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New Setting

LOCAL model, but ...

deterministic algorithms

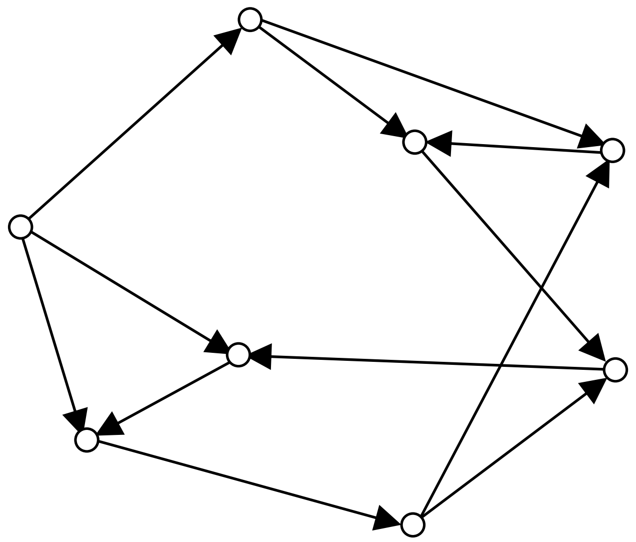
high-girth graphs

no unique identifiers

Sinkless Orientation

<u>Sinkless Orientation Problem:</u>

Orient the edges such that no node is a sink.



Sinkless Orientation

t-round algorithm
for sinkless orientation

$$\Downarrow$$

$(t-1)$-round algorithm
for sinkless orientation

[B., Fischer, Hirvonen, Keller, Lempiäinen, Rybicki, Suomela, Uitto, STOC'16]

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What about other problems?

[B., Fischer, Hirvonen, Keller, Lempiäinen, Rybicki, Suomela, Uitto, STOC’16]

Sinkless Orientation

t-round algorithm
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⇓

$(t-1)$-round algorithm
for sinkless orientation

What about other problems?

3-Coloring Cycles

[Linial, FOCS'87]
[Laurinharju, Suomela, PODC'14]

Even-Degree Weak 2-Coloring

[Balliu, Hirvonen, Olivetti, Suomela, PODC'19]

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The Old Speedup …

t-round algorithm
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$(t-1)$-round algorithm
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The New Speedup

Let Π_0 be any locally checkable problem. Then we can automatically find a locally checkable problem Π_1 such that

t-round algorithm
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$\Downarrow$

$(t-1)$-round algorithm
for sinkless orientation

t-round algorithm
for Π_0

$\Updownarrow$

$(t-1)$-round algorithm
for Π_1

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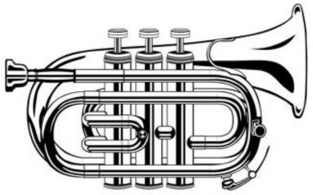
Π_0  Π_1

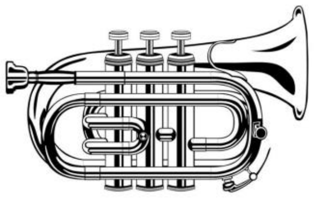
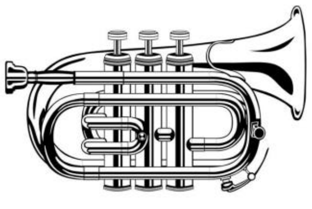
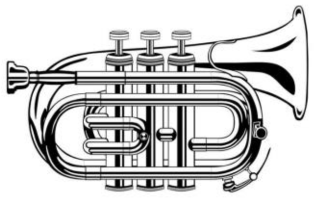
t-round algorithm for Π_0

$\Updownarrow$

$(t-1)$-round algorithm for Π_1

Problem	Π_0	Π_1
Complexity	$T(n, \Delta)$	$T(n, \Delta) - 1$

The New Speedup

Π_0  Π_1  Π_2  …

Problem	Π_0	Π_1	Π_2	…
Complexity	$T(n, \Delta)$	$T(n, \Delta) - 1$	$T(n, \Delta) - 2$	…

Applying the Speedup

Find the first problem in the sequence that can be solved in 0 rounds …

Problem	Π_0	Π_1	Π_2	…	Π_k
Complexity	$T(n,\Delta)$	$T(n,\Delta)-1$	$T(n,\Delta)-2$	…	0

Applying the Speedup

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Complexity	$T(n, \Delta)$	$T(n, \Delta) - 1$	$T(n, \Delta) - 2$	…	0

Π_0 has complexity k.

Where's the catch?

Increasingly complex problem descriptions!

Problem	Π_0	Π_1	Π_2	…	Π_k
Complexity	$T(n, \Delta)$	$T(n, \Delta) - 1$	$T(n, \Delta) - 2$	…	0

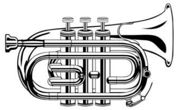
Π_0 has complexity k.

Simplifying the Problems

Much simpler description than Π_1

...

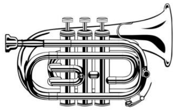
relax



Π'_1 → Π'_2

$T(n,\Delta) - ?$ $\quad$ $T(n,\Delta) - ? - 1$

relax



Problem	Π_0 →	Π_1
Complexity	$T(n,\Delta)$	$T(n,\Delta) - 1$

Lower Bounds

Π_k^*

0

Much simpler description than Π_1

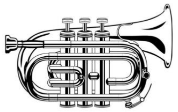
...

relax

Π_1' → Π_2'

$T(n,\Delta) - ?$ $T(n,\Delta) - ? - 1$

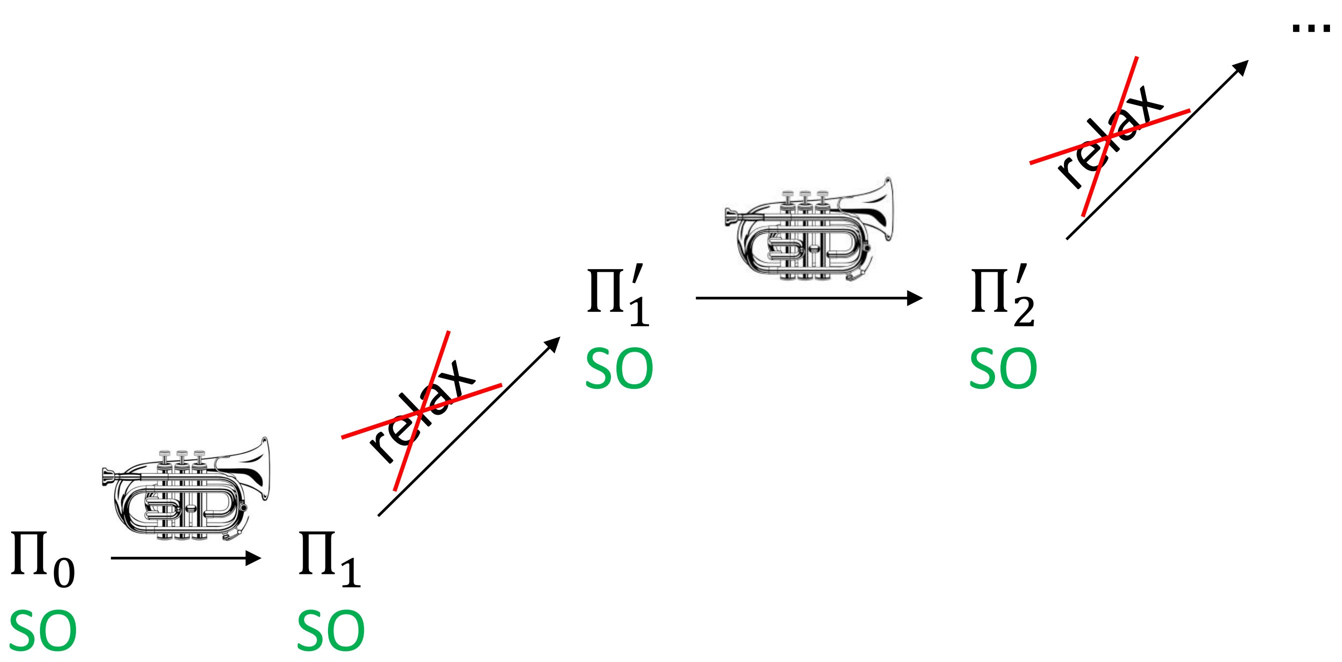
relax



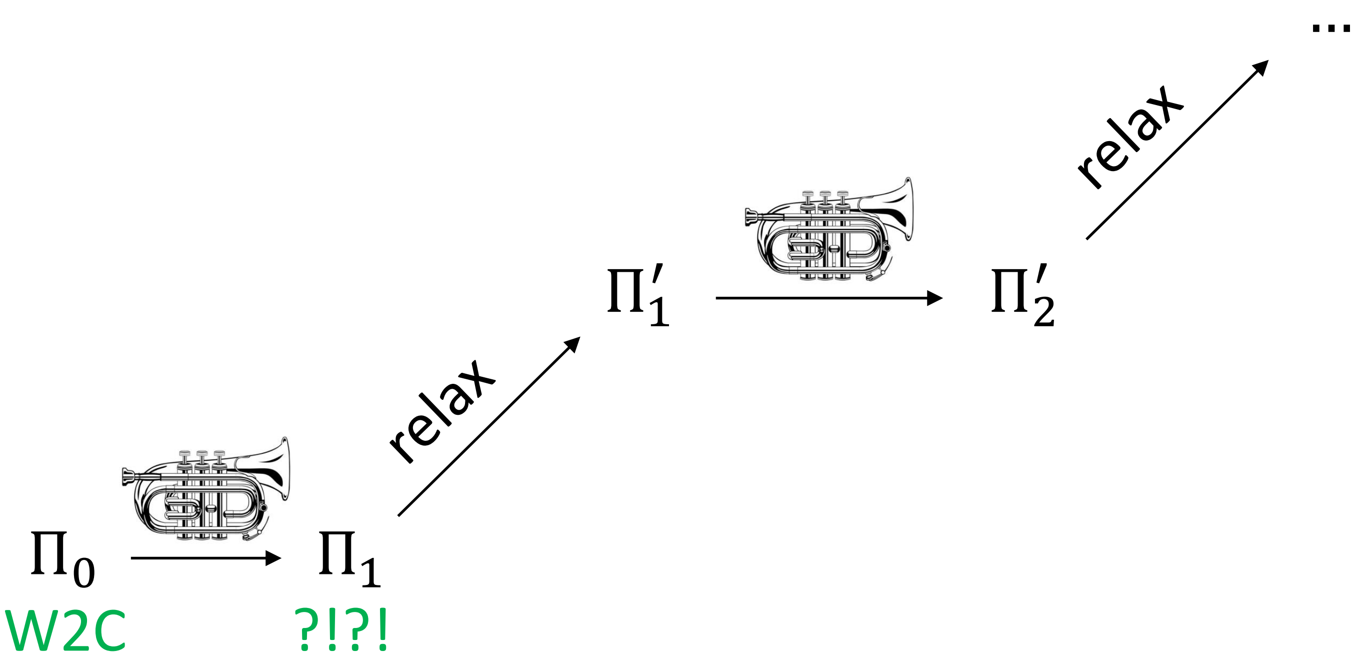
Problem	Π_0 →	Π_1
Complexity	$T(n,\Delta)$	$T(n,\Delta) - 1$

Π_0 has complexity at least k.

Sinkless Orientation



Odd-Degree Weak 2-Coloring



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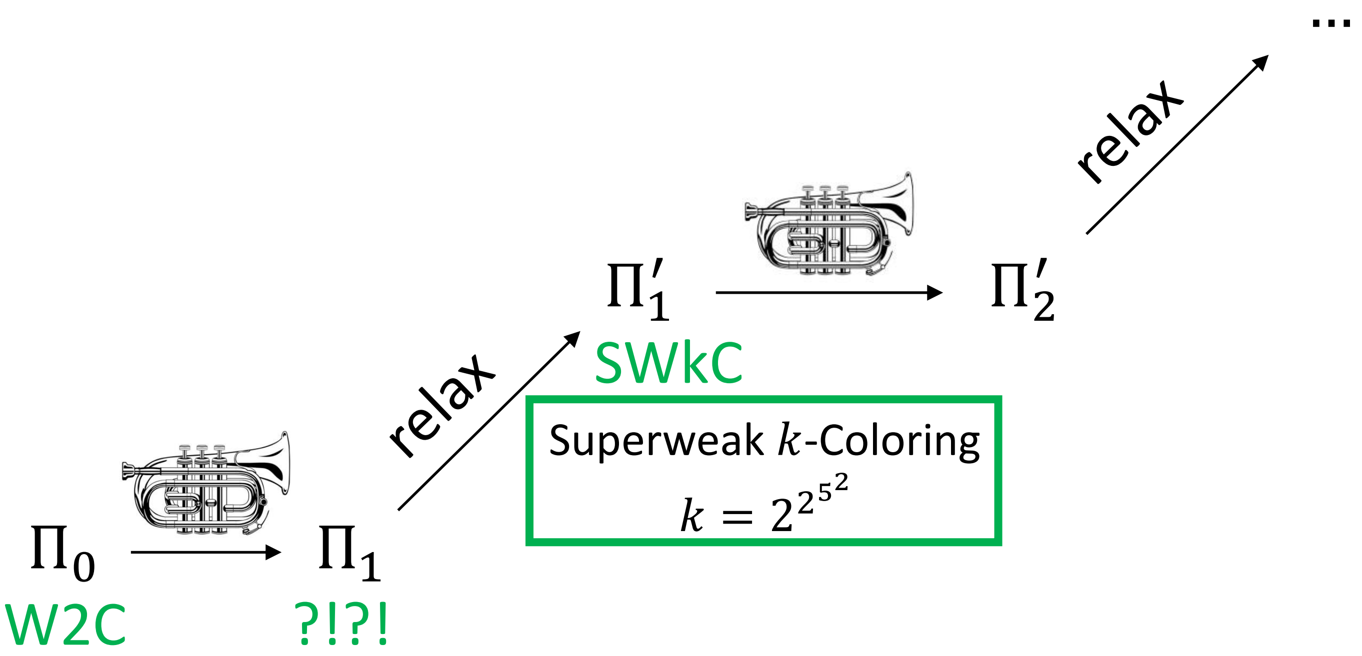

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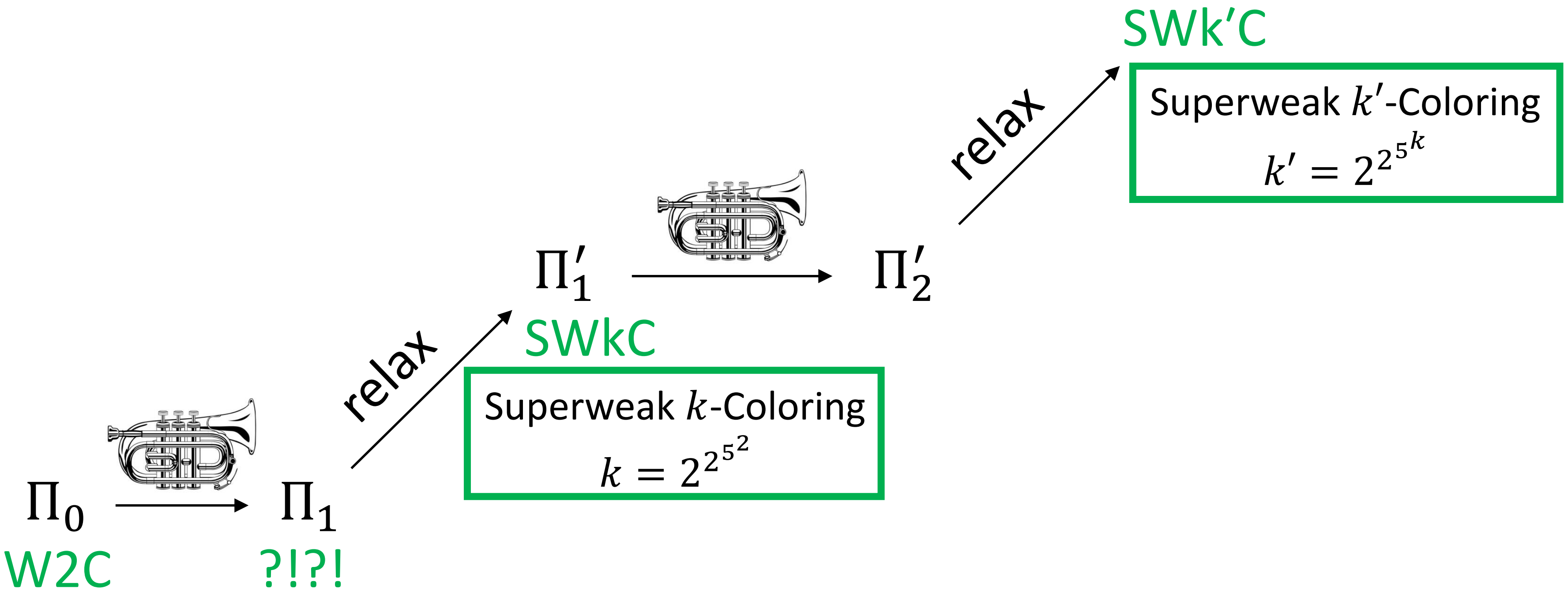

The Future

Unique IDs?

Other Problems?

Which Relaxations?

General Graphs?

Upper Bounds?

Better Lower Bounds for Vertex/Edge-Coloring?

Summary

Minimal Symmetry Breaking

Automatic Speedup Theorem

Even-Degree Weak 2-Coloring requires $\Omega(\log^* n)$ rounds.

minimality

new lower bounds

complexity gap for homogeneous LCLs

Odd-Degree Weak 2-Coloring requires $\Omega(\log^* \Delta)$ rounds.

validates

new lower bound technique

new lower bounds