Locality of weak and not-so-weak coloring

Dennis Olivetti
Aalto University, Finland

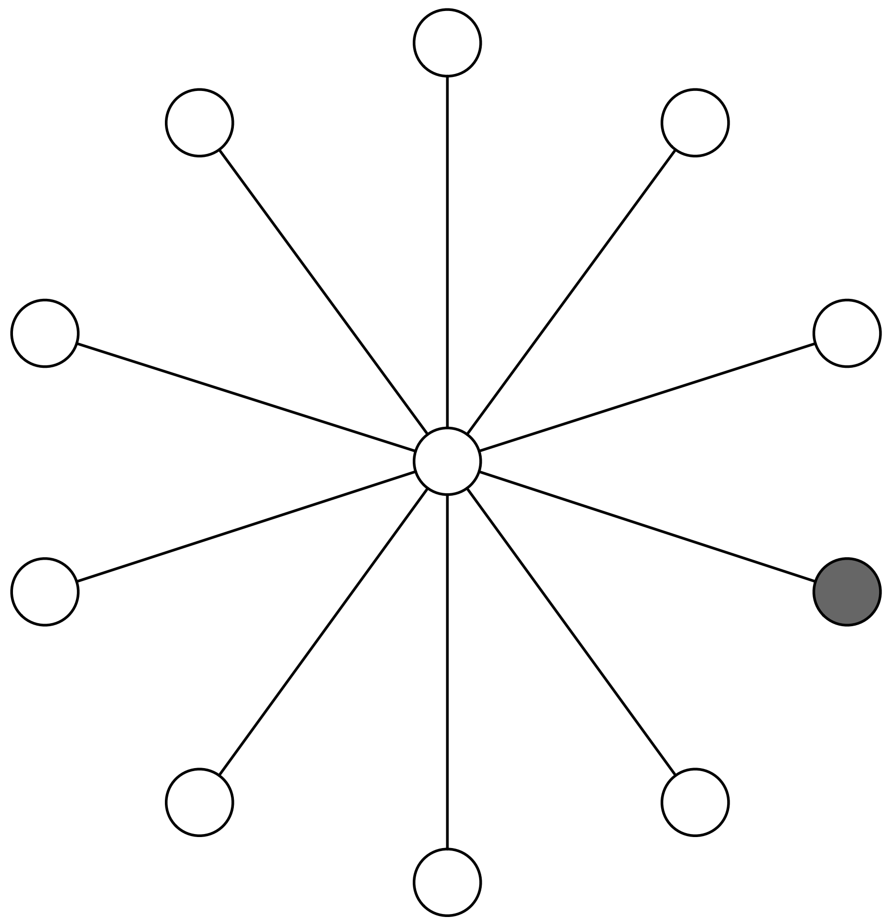
Joint work with

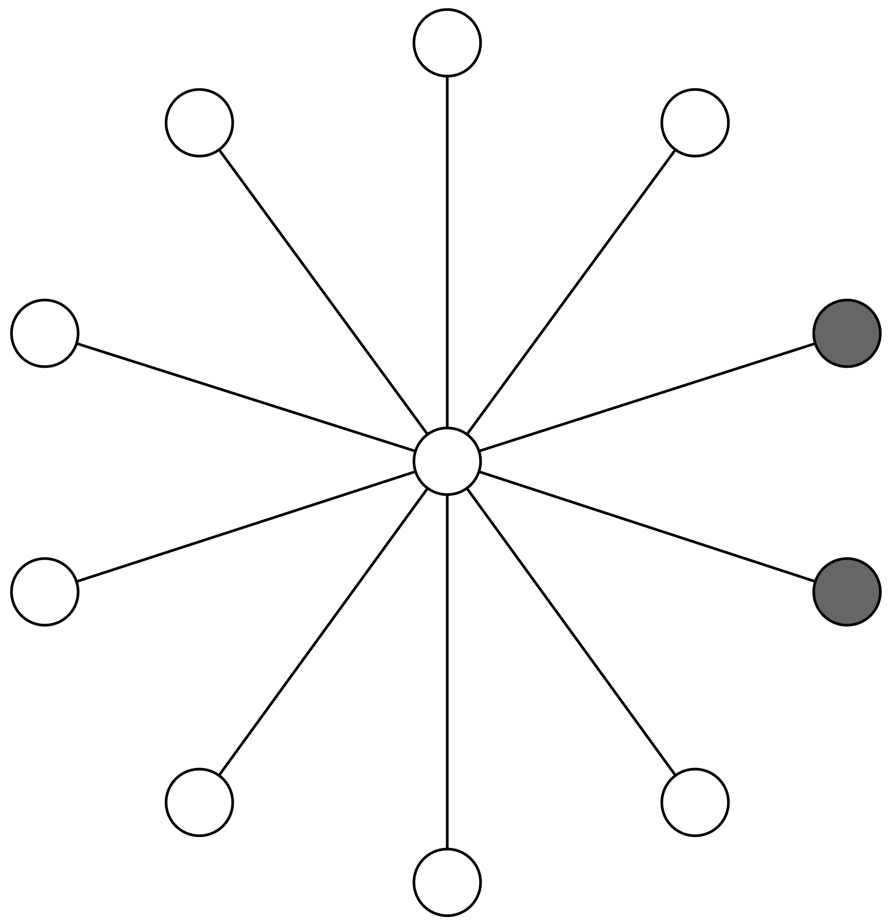
- **Alkida Balliu** · Aalto University
- **Juho Hirvonen** · Aalto University
- **Christoph Lenzen** · Max Planck Institute for Informatics
- **Jukka Suomela** · Aalto University

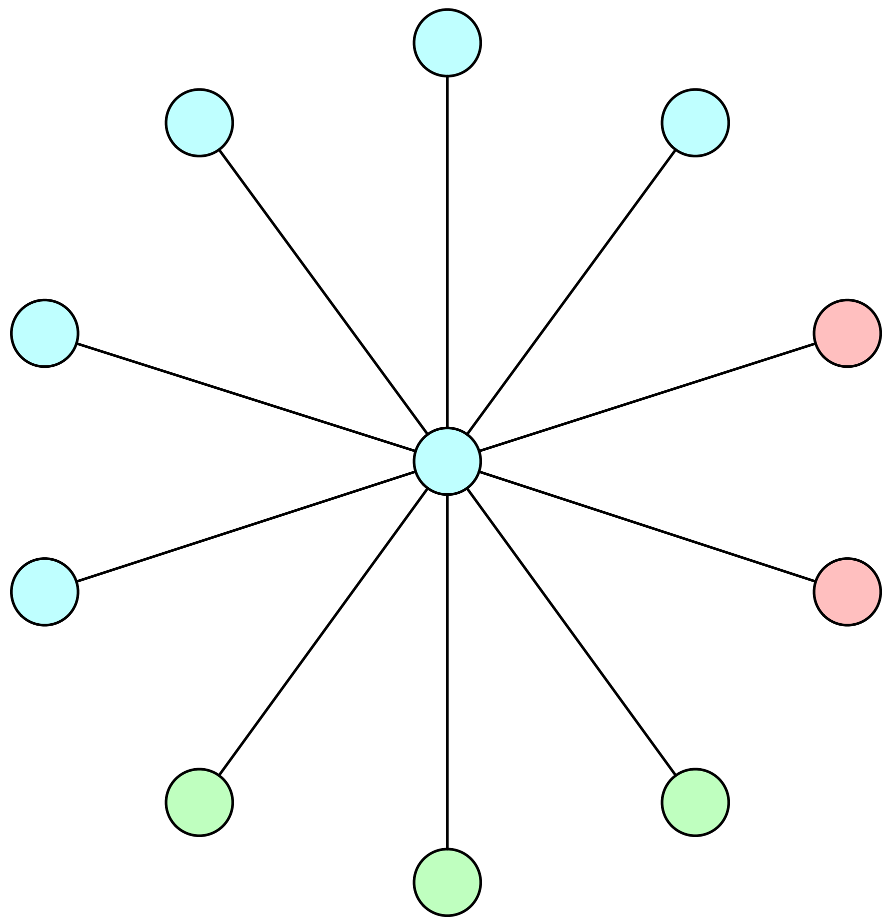
Hardness of minimal symmetry breaking in distributed computing **arXiv:1811.01643**

Locality of not-so-weak coloring **arXiv:1904.05627**

General Topic







LOCAL model

- Entities = **nodes**
- Communication links = **edges**
- Input graph = communication graph

LOCAL model

- Each node has a **unique identifier** from 1 to poly(n)
- **No bounds** on the computational power of the entities
- **No bounds** on the bandwidth

LOCAL model

- Round 0

LOCAL model

- Round 1

LOCAL model

- Round 2

LOCAL model

- After ***t* rounds**: knowledge of the graph up to **distance *t***
- Focus on **locality**

Weak 2-Coloring

2-coloring where each node has at least a neighbor of different color

Distributed Complexity of Weak 2-Coloring

- **$\Theta(\log^* \Delta)$** in odd-degree graphs [Naor and Stockmeyer 1995] [Brandt 2019]
- **$O(\log^* n)$** on general graphs
- **$\Omega(\log^* n)$** on cycles [Reduction from 3-coloring]
- **$\Omega(\log \log^* n)$** on regular trees [Naor and Stockmeyer 1995] [Chang and Pettie 2017]

The Ω(log log* n) lower bound

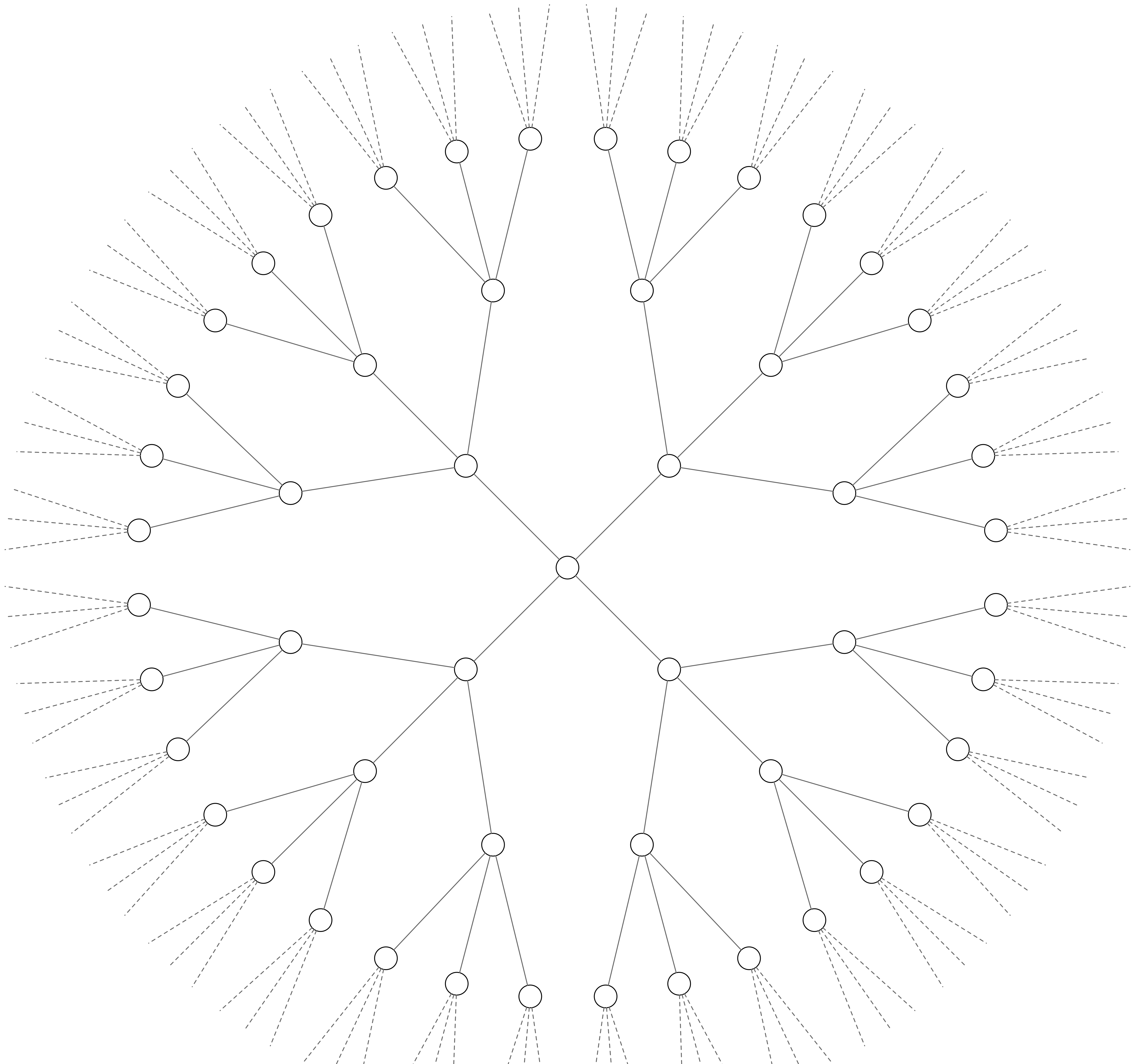
- Naor & Stockmeyer proved that any constant time algorithm for LCLs can be transformed to an order invariant algorithm
- On even regular trees, weak 2-coloring can not be solved by an order invariant algorithm
- Chang and Pettie lifted the gap up to **$\Omega(\log \log^* n)$**
- Both proofs use Ramsey theory
- Ramsey gives a lower bound on **volume, not distance**

Lower bound on cycles

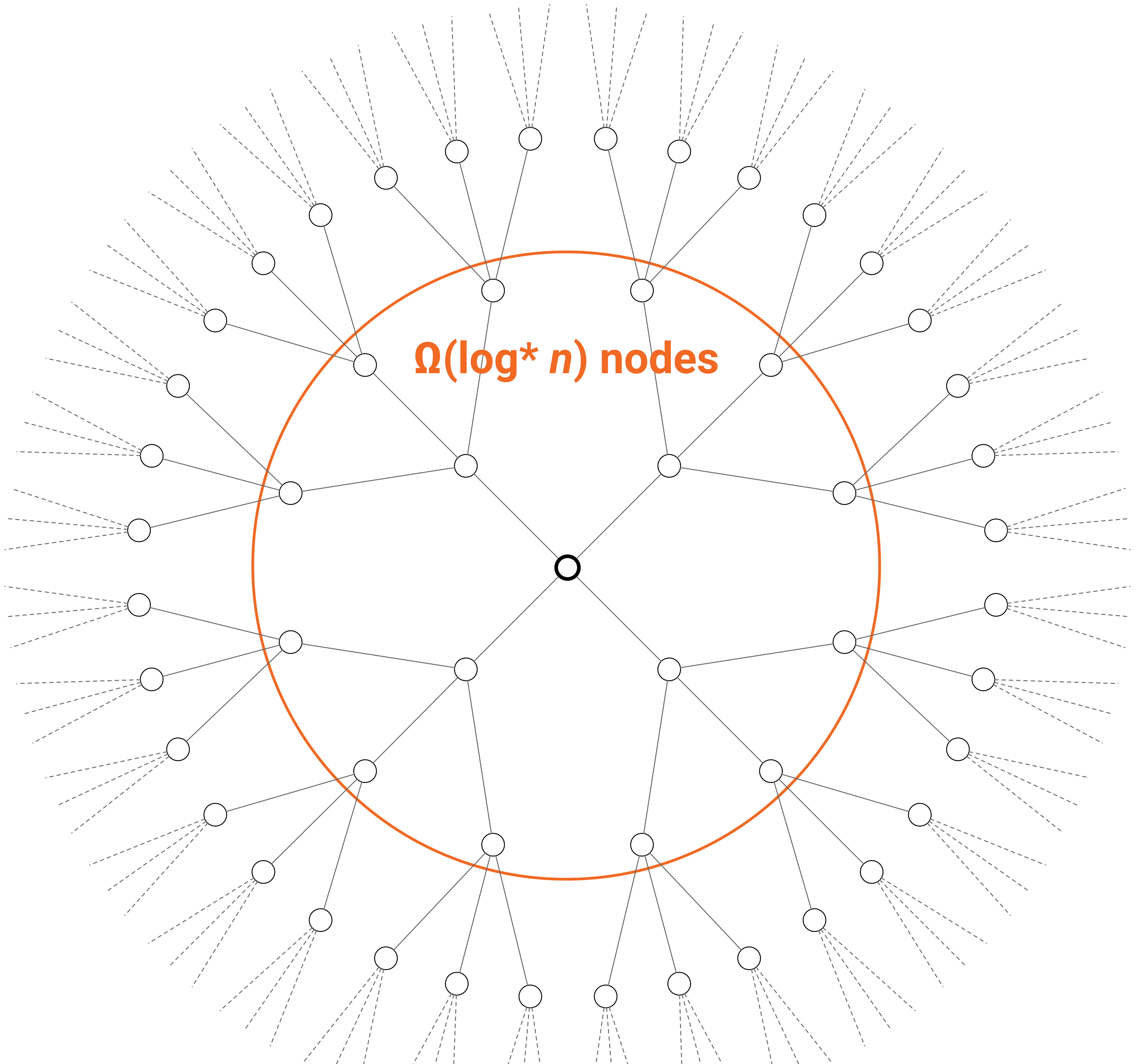
Lower bound on cycles

Lower bound on cycles

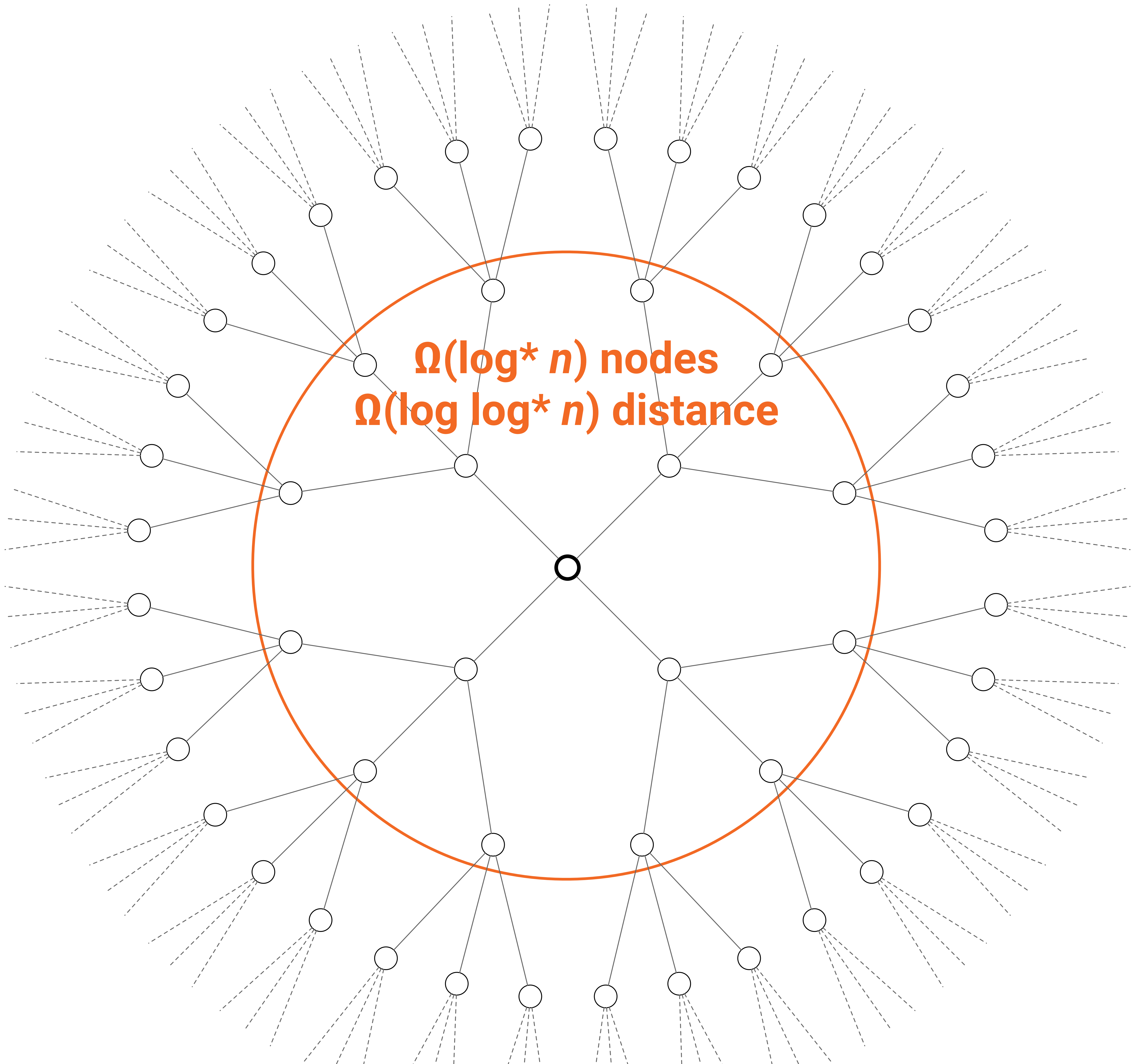
Lower bound on trees



Lower bound on trees



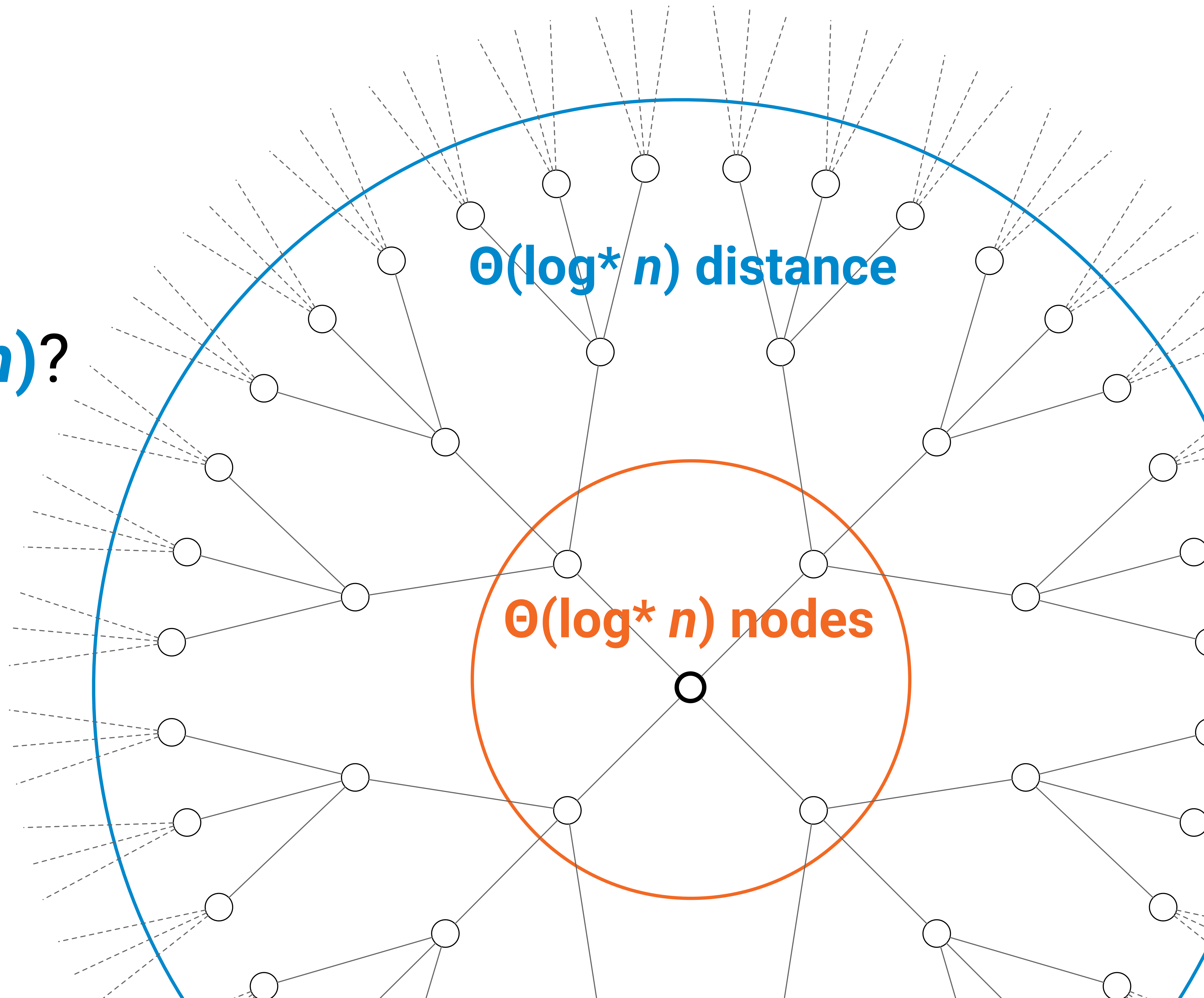
Lower bound on trees



Complexity in even degree regular graphs

- Lower bound of **Ω(log log* n)** distance and **Ω(log* n)** volume
- Upper bound of **O(log* n)** distance
- Is a **volume** of **O(log* n)** nodes enough?
- Or do we need to see at **distance Ω(log* n)**?

Is it easier to solve weak 2-coloring if we have many neighbors?

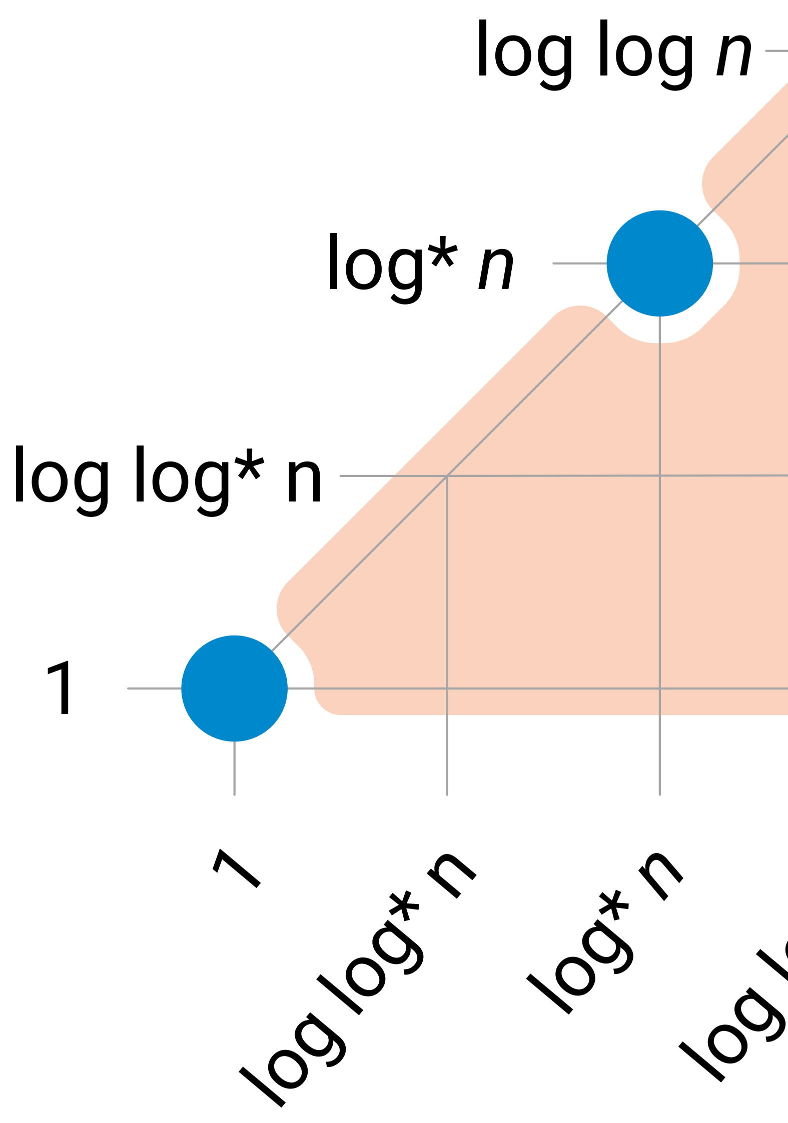


Our results

Weak 2-coloring requires **$\Omega(\log^* n)$** time in even-regular trees:

- For any constant even Δ
- Even if we allow randomization
- Even if identifiers are exactly in $\{1, \ldots, n\}$

Also, **weak 2-coloring** is the easiest possible non constant time "homogeneous LCL" problem



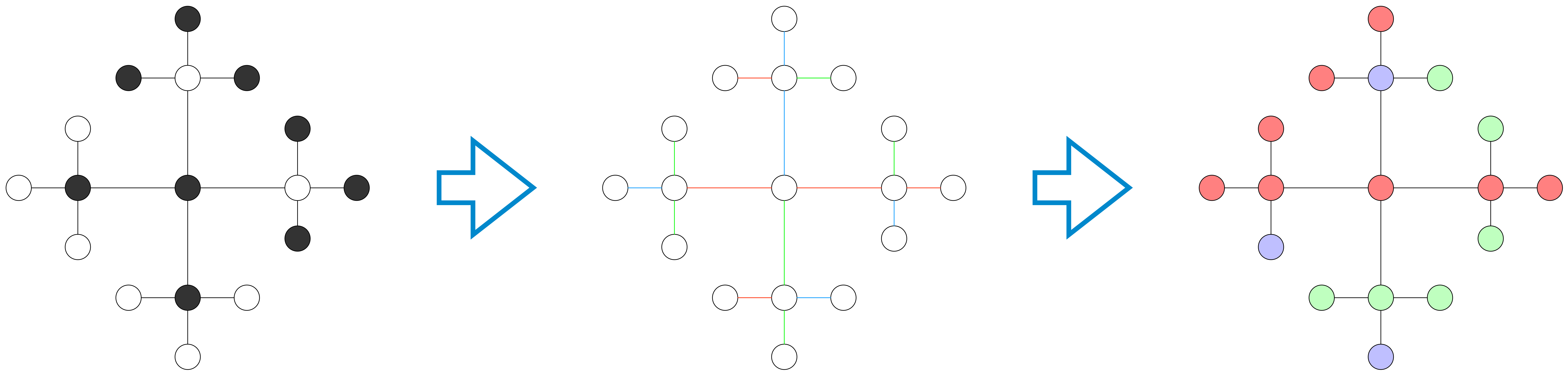
Speedup Simulation Technique

- Given:
  - an algorithm A_0 that solves problem P_0 in T rounds,
- We construct:
  - an algorithm A_1 that solves problem P_1 in $T-1$ rounds,
  - an algorithm A_2 that solves problem P_2 in $T-2$ rounds,
  - an algorithm A_3 that solves problem P_3 in $T-3$ rounds,
  - ...
  - an algorithm A_T that solves problem P_T in 0 rounds.
- We prove that P_T can not be solved in 0 rounds.

Speedup for Weak 2-Coloring

- Given an algorithm ***A*** that solves weak ***c*** coloring in ***T*** rounds, we construct an algorithm ***A'*** that solves "special" weak 2^{2c} edge coloring in ***T-1*** rounds

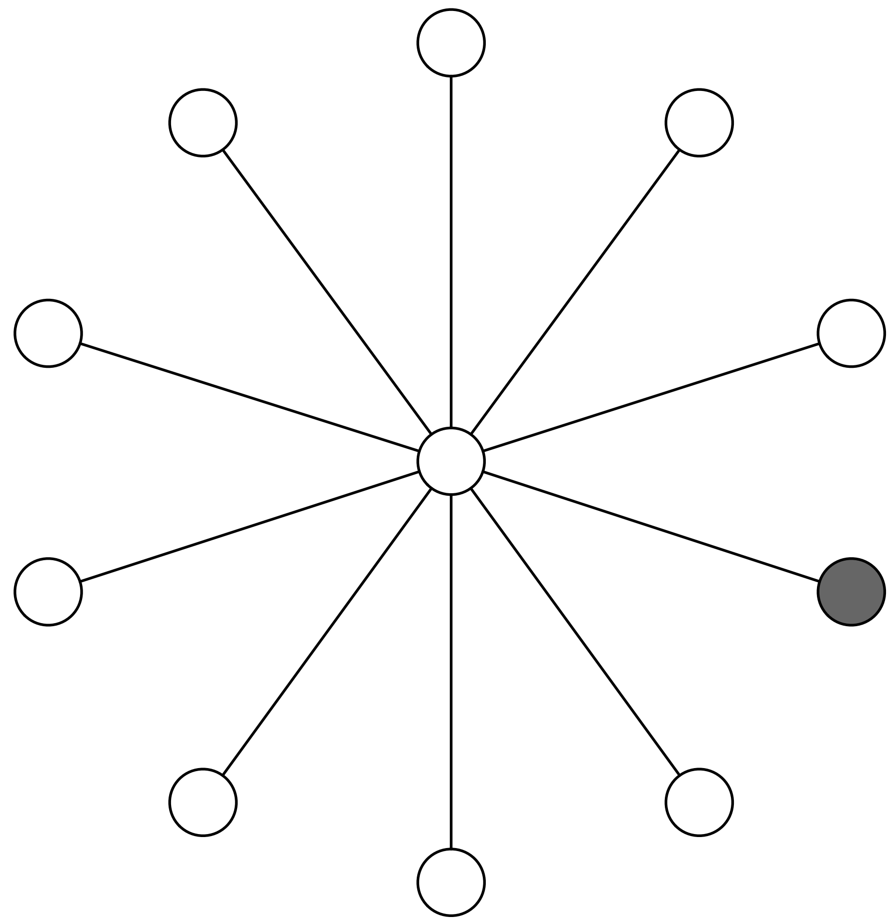
- Given an algorithm ***A*** that solves "special" weak ***c*** edge coloring in ***T*** rounds , we construct an algorithm ***A'*** that solves weak 2^{4c} coloring in ***T*** rounds



Beyond Weak 2-Coloring

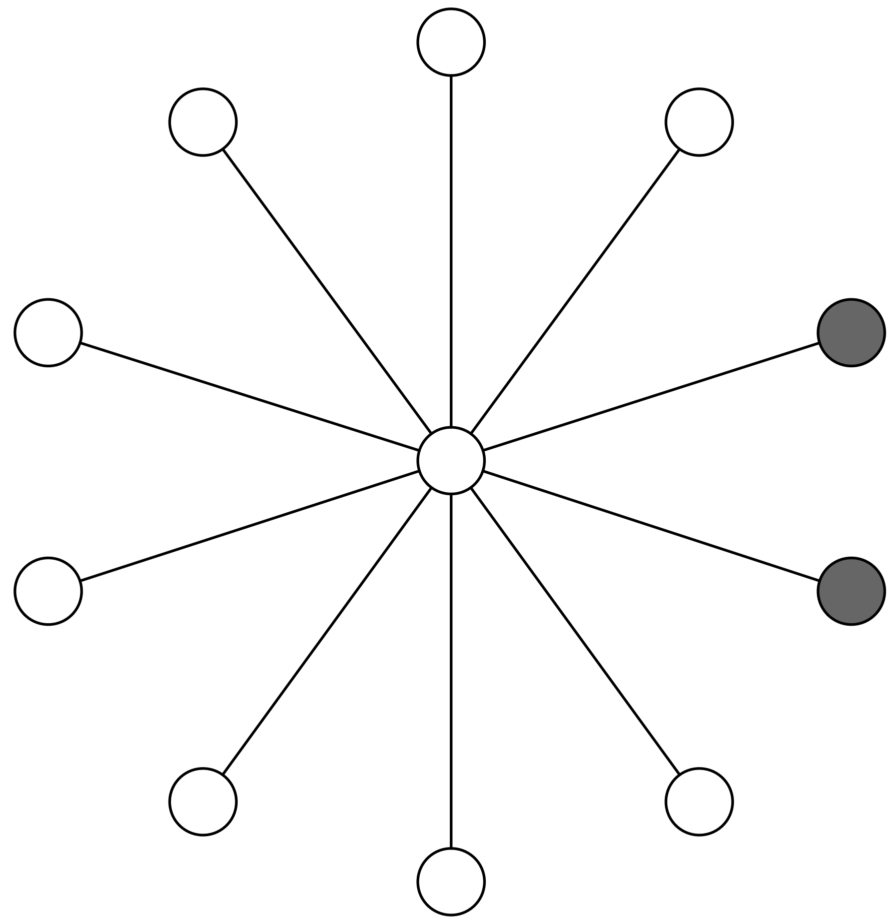
Weak 2-coloring

- 2-color the nodes such that each node has at least 1 neighbor of different color



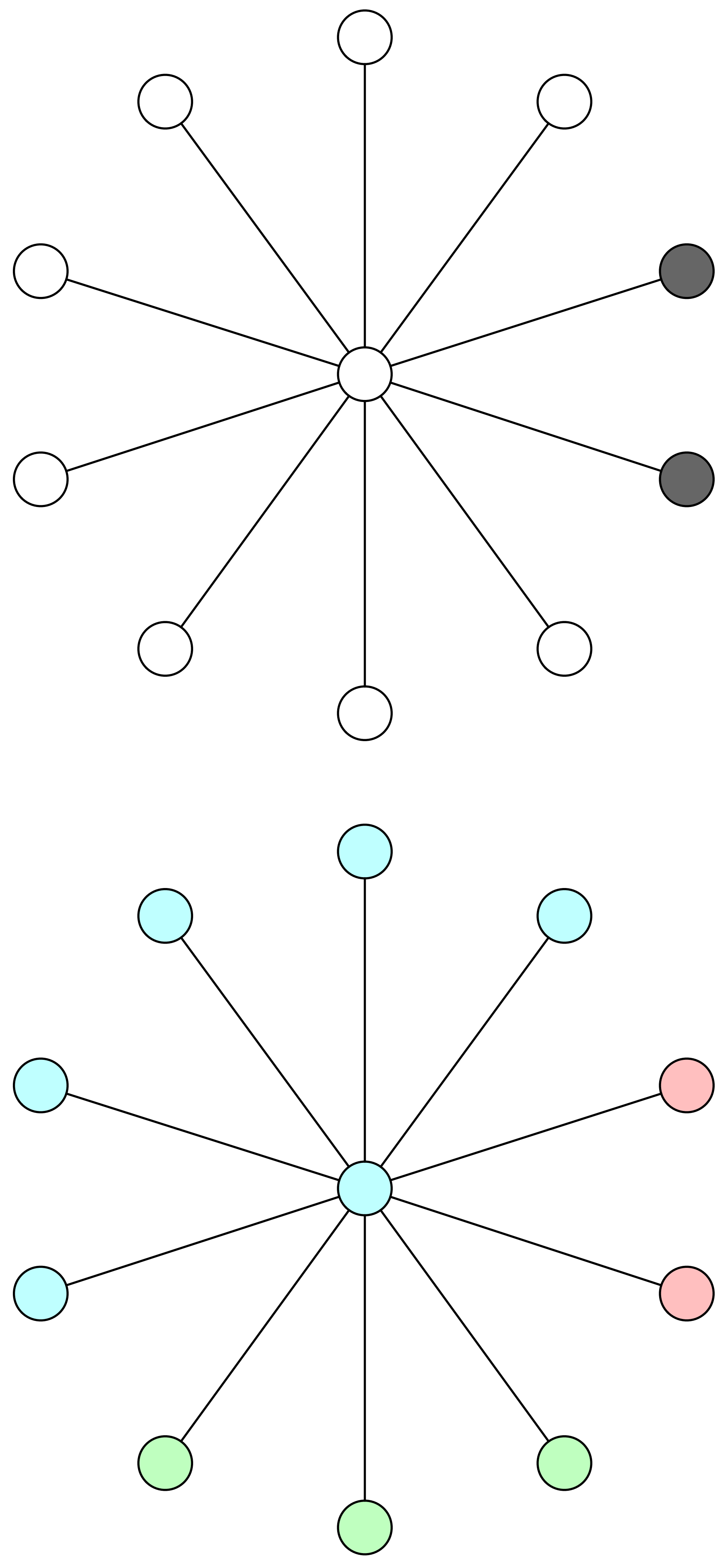
2-Partial 2-Coloring

- 2-color the nodes such that each node has at least 2 neighbors of different color



Our results

- **2-partial 2-coloring** requires:
  - **$\Omega(\log n)$** for any constant **$\Delta \geq 2$**
- **k-partial 3-coloring** requires:
  - **$\Omega(\log n)$** for **$\Delta = k$**
  - **$O(\log^* n)$** for **$\Delta \gg k$**



Conclusions

- **Weak 2-Coloring** requires **$\Theta(\log^* n)$** time on **Δ** regular trees
- Requiring **2 neighbors** of different color, instead of just 1, makes the problem much harder, **$\Omega(\log n)$**, even if **$\Delta = 1000$**
- Open problem:
  - **3**-partial **3**-coloring on **3**-regular graphs is **$\Omega(\log n)$** (it is Δ-coloring)
  - **3**-partial **3**-coloring on **5**-regular graphs is **$O(\log^* n)$**
  - What is the complexity of **3**-partial **3**-coloring on **4**-regular graphs?

Thank you!