Albert-Ludwigs-Universität Freiburg
Dept. of Computer Science
Algorithms and Complexity

# Distributed Coloring in Fully Connected Communication Networks

**Bachelor or Master Thesis (or Project)**

## Project Description

Given a graph $G = (V, E)$, the vertex coloring problem asks for assigning a color to each node $v \in V$ such that no two neighboring nodes are assigned the same color and such that the total number of colors used is as small as possible. Traditionally, when studying a graph problem such as vertex coloring in a distributed setting, one assume that the graph $G$ represents the communication network: each node of $G$ is an individual computing device and the edges represent bidirectional communication channels over which messages can be exchanged. Typically, one considers a synchronous communication model, where time is divided into synchronous rounds and in each round, every node $v \in V$ can send a message to every neighboring node in $G$. The objective then is to get an algorithm that solves the given problem in as few rounds as possible.

Recently, there has also been an increased interest in communication models where every node can directly talk to every other nodes. Such models are for example closely related to what is needed to understand the cost of solving graph problems on a cluster of computers. The most widely studied communication model of this kind is the so-called *congested clique* model. Also here, every node $v \in V$ of $G$ is a node of the network. However, in each round, a node $v$ can not only send a message to each of its neighbors, but also to every other node in the network. Each of these messages needs to consist of at most $O(\log n)$ bits, where $n$ is the number of nodes of $G$. In such a very powerful communication model, many problems (such as standard variants of the distributed coloring problem) can be solved in a constant number of communication rounds. The goal of this project is to look at weaker variants of such a clique model. One could for example introduce a parameter $k \in \{1, \dots, n-1\}$ and require that each node can only send and receive at most $k$ $O(\log n)$-bit messages in each communication round. These $k$ messages can however be sent to and received from arbitrary $k$ other nodes in the graph. What is the complexity of graph coloring as a function of $k$? What about related graph problems? One can also look at other relaxations of the congested clique model or a similar relaxations of other related communication models.

### Requirements

- mathematical maturity, interest in mathematical questions
- algorithm theory and/or distributed systems (network algorithms) lecture (or comparable lectures) are an advantage

### Contact

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