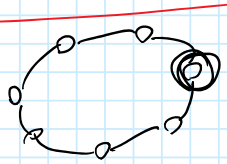
Leader Election (LE)

Thursday, May 15, 2014 12:19 PM

Problem:
- all nodes v decide to be a leader or not
- $\exists_1$ leader

Here: **rings**



- undecided
- leader
- not leader

Anonymous network: nodes do not have ID_v

Claim: LE is not possible in anonymous networks (det.)

Proof: $i=0$: all undecided $\Rightarrow$ all have same state

$i \rightsquigarrow i+1$: all move to same new state (synchronous)

synchronous $\subset$ asynchronous !!

$\Rightarrow$ also not possible in asynchronous networks

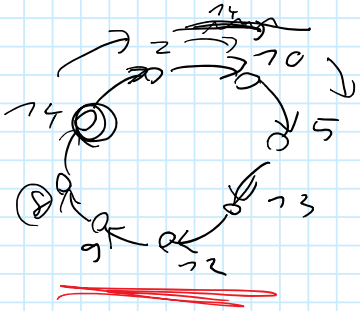
Def.: non-uniform: $\Leftrightarrow$ nodes know n

uniform: $\Leftrightarrow$ otherwise

$n \geq 2$: also not possible in non-uniform networks.

with ID_v (unique)

leader: v with max ID (or min ID)



Clockwise

start sending ID to clockwise neighbor

upon receive from w: ($ID_{max} := ID_v$)

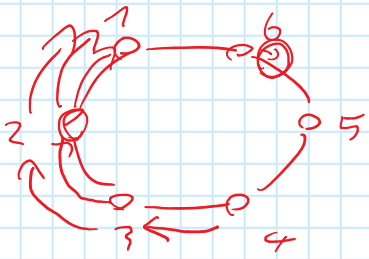
$ID_w > ID_{max}$ $\Rightarrow$ set "not leader", $ID_{max} = ID_w$, forward $ID_w = ID_{max}$

$ID_w = ID_v$ $\Rightarrow$ set "leader"

Correctness: trivial
(all nodes see ID_{max} and then decide)

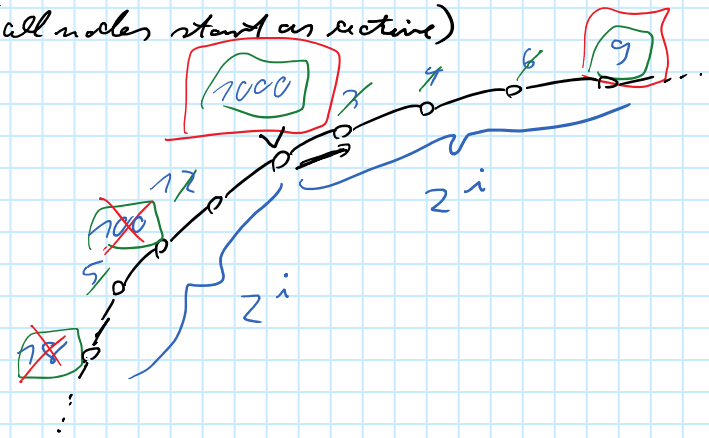
time: $O(n)$

msg. compl.: $O(n^2)$



Hirschberg-Sinclair

phases: $i = 0, 1, 2, 3, 4, \ldots$
(all nodes start as active)



"probe message" with time-to-live counter (TTL) $= 2^i$

phases: $\lceil \log n \rceil = O(\log n)$

time/phase: $\sim 2 \cdot 2^i = O(2^i)$

Actual time: $2 \cdot [2^0 + 2^1 + 2^2 + \ldots + 2^{\lceil \log n \rceil}]$

geometric series

$\leq 2 \cdot 2 \cdot 2^{\lceil \log n \rceil} = O(n)$

msg. compl.

phase i: # active nodes:

$\checkmark$ active in phase i: $\checkmark$ eliminated
other active nodes in phase i $\rightarrow$

$\Rightarrow$ i distance 2^{i-1}

$\leq \frac{n}{2^{i-1}}$

msgs / active node : $4 \cdot 2^i$

$\Rightarrow \frac{n}{2^{i-1}} \cdot 4 \cdot 2^i = O(n)$ per phase!

$\Rightarrow O(n \log n)$ messages

Can we do better?

Lower Bounds

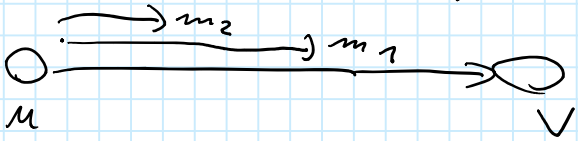
$\forall$ asynchronous algorithms:

$\exists$ execution that causes $\Omega(n \log n)$ messages being sent

Execution: list of (receive) events

simplicity: no two events happen at the same time

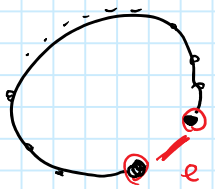
- FIFO:

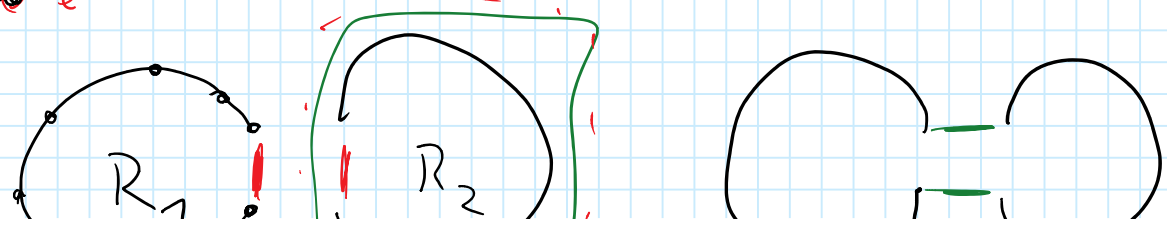


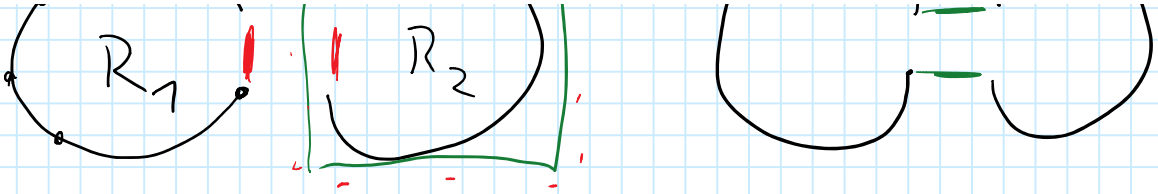
- msg ID gets to become leader
- uniform
- spontaneous wake-up

Open Schedule

$\exists$ e s.t. no message over e has been received so far






R1
R2

For nodes scenario of ring R_1 is not distinguishable from scenario with ring $R = "R_1 \cup R_2"$