

Network Algorithms, Summer Term 2015

Problem Set 7

hand in by June 16, 2015

Exercise 1: Coloring Rings

In Chapter 1, we proved that a ring can be colored with 3 colors in $\log^* n + O(1)$ rounds. Clearly, a ring can only be (legally) colored with 2 colors if the number of nodes is even.

1. Prove that, even if the nodes in a directed ring know that the number of nodes is even, coloring the ring with 2 colors requires $\Omega(n)$ rounds!¹

Since coloring a ring with 2 colors apparently takes a long time, we again resort to the problem of coloring rings using 3 colors.

- b) Assume that a *maximal independent set* (MIS) has already been constructed on the ring, i.e., each node knows whether it is in the independent set or not. Give an algorithm to color the ring with 3 colors in this scenario! What is the time complexity of your algorithm? Deduce from this a lower bound for computing a MIS!

¹As in the lecture, the message size and local computations are unbounded and all nodes have unique identifiers from 1 to n .