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Network Algorithms, Summer Term 2016
Problem Set 9

hand in by Wednesday, July 6, 2016

Counting with Asynchronous Wake-up (Start)

Recall the counting problem in (edge-dynamic) dynamic networks presented in the lecture. Communication is synchronous, message size is arbitrary, and each node has a unique identifier. We want the all nodes to learn the number of nodes (which is assumed to be n).

We assume that for any two consecutive rounds r and $r+1$, the (static) graph $(V, E(r) \cap E(r+1))$ is connected. Now we drop the assumption from the lecture that all the nodes wake up at the same time. Instead, some node $u \in V$ wakes up by itself, while all other nodes start executing the respective algorithm when they receive their first message.

1. Show that anything that can be done if a single node u starts the computation and all other nodes are woken up when they receive their first messages, can also be done if the nodes wake up spontaneously, without receiving a message. Note that nodes still wake up upon receiving the first message if they are not awake by that time.

2. Devise an algorithm that receives an input k and let u decide whether $k \leq n$ or $k > n$ within $O(k)$ rounds.

   Hint: Make u wake up all the nodes and collect all the identifiers assuming that we have less than k nodes. With a little extra time, one will see more than k identifiers if $n > k$.

3. Use your algorithm as a subroutine for an algorithm that determines n up to a factor 2 in $O(n)$ time. Can n also be determined exactly?