Albert-Ludwigs-Universität
Institut für Informatik
Prof. Dr. F. Kuhn

Exam Theoretical Computer Science - Bridging Course

Wednesday, August 17, 2016, 10:00-12:00

Name: ....

Matriculation Nr.: ....

Signature: ....

Do not open or turn until told so by the supervisor!

- Write your name and matriculation number on the cover page of the exam and sign the document! Write your name on all sheets!
- Your signature confirms that you have answered all exam questions without any help, and that you have notified exam supervision of any interference.
- Write legibly and only use a pen (ink or ball point). Do **not** use **red**! Do **not** use a pencil!
- There are 6 problems (with several questions per problem) and there is a total of 120 points. At least 40% are needed to pass the exam, and 80% will net you the best grade.
- Only one solution per question is graded! Make sure to strike out any solutions that you do not want to be considered!
- **Explain your solution!** Just writing down the end result is not sufficient unless otherwise indicated. Detailed steps can also help you get more points in case your final result is incorrect.
- This is an **open book exam**. However, only printed or hand-written material is allowed, no electronic devices are allowed.

Problem	Achieved Points	Max Points
1		20
2		20
3		15
4		15
5		28
6		22
Total		120

Problem 1: Automata and Languages (20 points)

Consider the following language $L = \{w \mid w$ starts with arbitrary number (including zero) of a, followed by arbitrary number (including zero) of b, and then ends with at least one $a\}$. The alphabet for L is $\{a, b\}$. Answer the following questions:

(a) (4 points) Give a regular expression that generates L.

(b) (7 points) Construct a *three-state* NFA that recognizes L.

(c) (9 points) Construct a DFA which recognizes L. (For example, you can covert the NFA from (b), if you believe it is correct.)

Problem 2: Automata and Languages (20 points)

Let the be alphabet $\Sigma = \{0, 1\}$. For each of the following languages, answer if it is a context-free language (CFL) or not. You need to prove your answer as well.

(a) (5 points) $L_1 = L^*$, where L is a CFL.

(b) (6 points) $L_2 = \{w \mid w = w^{\mathcal{R}}$, which means w is a palindrome$\}$. (Note that an empty string is also a palindrome.)

(c) (9 points) $L_3 = \{w \mid w = w^{\mathcal{R}}$ and w contains an equal number of 0s and 1s$\}$.

Problem 3: Computability (15 points)

Prove that the following languages are decidable.

(a) (5 points) $L_1 = \overline{L}$, where L is a decidable language. (That is, L_1 is the complement of L.)

(b) (10 points) $L_2 = \{\langle R \rangle \mid R$ is a regular expression describing a language containing at least one string w that has 00 as a substring$\}$.

Problem 4: Big-O Notation (15 points)

Determine the relationship between the functions $f(n)$ and $g(n)$ for the following three examples. That is, state whether it holds that $f(n) = O(g(n))$, $g(n) = O(f(n))$, or if both these statements are true (which would imply $f(n) = \Theta(g(n))$). Explain your answers!

(a) (4 points) $f(n) = n^{100}, \quad g(n) = 2^n$

(b) (5 points) $f(n) = \log_{10} n, \quad g(n) = 100 \log_2 n$

(c) (6 points) $f(n) = (2n)!, \quad g(n) = n^n$

Problem 5: Complexity (28 points)

(a) (12 points) An instance of SET-COVER is given by a set of elements U, a collection of m sets $S_1, S_2, \cdots, S_m$ such that each set S_i is a subset of U, and a positive integer k. The question is, can you find a collection $\mathcal{C}$ containing at most k of the subsets S_i of U such that taken together, they "cover" all elements in U? In other words, is there a set $\mathcal{C} \subseteq \{S_1, \ldots, S_m\}$ such that $|\mathcal{C}| \leq k$ and $\bigcup_{S_i \in \mathcal{C}} S_i = U$? Prove that SET-COVER is NP-complete. (Hint: you may want to consider the connection between SET-COVER and VERTEX-COVER.)

(b) (16 points) Let T-SAT $= \{\langle\phi\rangle \mid \phi$ has at least three satisfying assignments$\}$. Show that T-SAT is NP-complete.

Problem 6: Logic (22 points)

(a) (8 points) Given the knowledge base $KB = \{\neg p \vee q \to r, s \vee \neg q, \neg t, p \to t, \neg p \wedge r \to \neg s\}$, derive $\neg q$ from KB.

(b) (14 points) A *predicate* is a sentence that contains a finite number of variables and becomes a statement when specific values are substituted for the variables. (For example, "$x^2 > x$" is a predicate.) The domain of a predicate variable is the set of all values that may be substituted in place of the variable.

Let $P(x)$ and $Q(x)$ be predicates and suppose D is the domain of x. For each of the following pair of statements, determine whether the two statements are equivalent. If the two statements are equivalent, no explanation is needed, if they are not equivalent, you need to give a counterexample.

- $\forall x \in D, (P(x) \wedge Q(x))$, and $(\forall x \in D, P(x)) \wedge (\forall x \in D, Q(x))$.
- $\exists x \in D, (P(x) \wedge Q(x))$, and $(\exists x \in D, P(x)) \wedge (\exists x \in D, Q(x))$.
- $\forall x \in D, (P(x) \vee Q(x))$, and $(\forall x \in D, P(x)) \vee (\forall x \in D, Q(x))$.
- $\exists x \in D, (P(x) \vee Q(x))$, and $(\exists x \in D, P(x)) \vee (\exists x \in D, Q(x))$.