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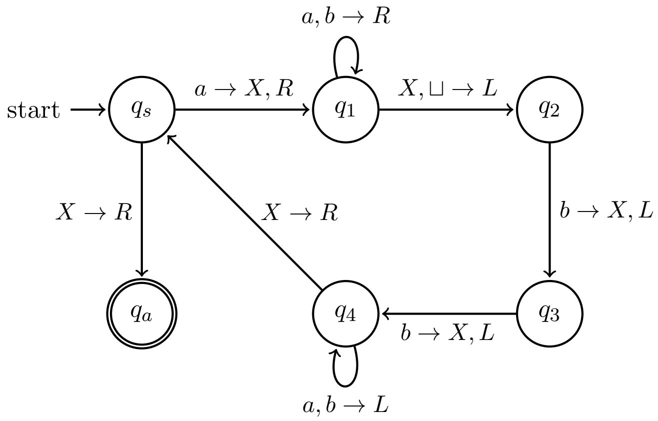
Theoretical Computer Science - Bridging Course
Summer Term 2017
Exercise Sheet 5 – Sample Solution

Exercise 1: Turing Machines Given as State Diagrams (2+3+2 points)

Consider the Turing machine M over the alphabet $\Sigma = \{a, b\}$, which is given via the following state diagram. *Note: The blank symbol* $\sqcup \in \Gamma$ *represents an empty cell on the tape.*



(a) Simulate M with input $s_1 = aabbbb$ on its tape until it halts. Give the configurations that M passes through. You may omit configurations where no symbol is replaced. State whether $s_1 \in L$.

(b) Simulate M with input $s_2 = aabbb$ on its tape until it halts. Give the configurations that M passes through. You may omit configurations where no symbol is replaced. State whether $s_2 \in L$.

(c) Give a description of the language $L(M)$ that M recognizes in the form of a set.

Sample Solution

(a) M run on input s_1 yields the following configurations:

$\sqcup$	$q_s\ a$	a	b	b	b	b	$\sqcup$
$\sqcup$	X	q_1a	b	b	b	b	$\sqcup$
$\sqcup$	X	a	q_1b	b	b	b	$\sqcup$
$\sqcup$	X	a	b	q_1b	b	b	$\sqcup$
$\sqcup$	X	a	b	b	q_1b	b	$\sqcup$
$\sqcup$	X	a	b	b	b	q_1b	$\sqcup$
$\sqcup$	X	a	b	b	b	b	$q_1\sqcup$
$\sqcup$	X	a	b	b	b	q_2b	$\sqcup$
$\sqcup$	X	a	b	b	q_3b	X	$\sqcup$
$\sqcup$	X	a	b	q_4b	X	X	$\sqcup$
$\sqcup$	X	a	q_4b	b	X	X	$\sqcup$
$\sqcup$	X	q_4a	b	b	X	X	$\sqcup$
$\sqcup$	q_4X	a	b	b	X	X	$\sqcup$
$\sqcup$	X	q_sa	b	b	X	X	$\sqcup$
$\sqcup$	X	X	q_1b	b	X	X	$\sqcup$
$\sqcup$	X	X	b	q_1b	X	X	$\sqcup$
$\sqcup$	X	X	b	b	q_1X	X	$\sqcup$
$\sqcup$	X	X	b	q_2b	X	X	$\sqcup$
$\sqcup$	X	X	q_3b	X	X	X	$\sqcup$
$\sqcup$	X	q_4X	X	X	X	X	$\sqcup$
$\sqcup$	X	X	q_sX	X	X	X	$\sqcup$
$\sqcup$	X	X	X	q_aX	X	X	$\sqcup$

(b) M run on input s_2 yields the following configurations:

$\sqcup$	$q_s\ a$	a	b	b	b	$\sqcup$
$\sqcup$	X	q_1a	b	b	b	$\sqcup$
$\sqcup$	X	a	q_1b	b	b	$\sqcup$
$\sqcup$	X	a	b	q_1b	b	$\sqcup$
$\sqcup$	X	a	b	b	q_1b	$\sqcup$
$\sqcup$	X	a	b	b	b	$q_1\sqcup$
$\sqcup$	X	a	b	b	q_2b	$\sqcup$
$\sqcup$	X	a	b	q_3b	X	$\sqcup$
$\sqcup$	X	a	q_4b	X	X	$\sqcup$
$\sqcup$	X	q_4a	b	X	X	$\sqcup$
$\sqcup$	q_4X	a	b	X	X	$\sqcup$
$\sqcup$	X	q_sa	b	X	X	$\sqcup$
$\sqcup$	X	X	q_1b	X	X	$\sqcup$
$\sqcup$	X	X	b	q_1X	X	$\sqcup$
$\sqcup$	X	X	q_2b	X	X	$\sqcup$
$\sqcup$	X	q_3X	X	X	X	$\sqcup$

(c) $L(M) = \{a^n b^{2n} \mid n \in \mathbb{N}\}$.

Exercise 2: Formal Definition of Turing Machines (5 points)

Let M be a Turing machine over the alphabet $\Sigma = \{0, 1\}$, with state set $Q = \{q_0, q_1, q_2, q_3, q_4, \overline{q}\}$, starting state q_0, accepting state $\overline{q}$ and transition function δ given via the following table

	0	1	$\sqcup$
q_0	$(q_0, 0, R)$	$(q_0, 1, R)$	$(q_1, \sqcup, L)$
q_1	$(q_2, \sqcup, R)$	$(q_3, \sqcup, R)$	$(\overline{q}, \sqcup, R)$
q_2	$(q_4, 0, L)$	$(q_4, 0, L)$	$(q_4, 0, L)$
q_3	$(q_4, 1, L)$	$(q_4, 1, L)$	$(q_4, 1, L)$
q_4	$(q_4, 1, R)$	$(q_4, 0, R)$	$(q_1, \sqcup, L)$

The above Turing machine accepts every word, i.e., it recognizes the language Σ^*. Describe the behaviour of M on an arbitrary input $w \in \{0, 1\}^*$ to explain how the content of the tape after the computation is related to the input word. Do not forget to explain **how** the Turing machine achieves its goal (in particular describe the role of q_2 and q_3) - it is not sufficient (to gain full points) to only name the final result.

Sample Solution:

The Turing machine moves the input one position to the right.
The states have the following meaning: Starting in q_0 the head is moved to the right until we find a blank, then we move one position to the left and change to q_1. In q_1 we either read a number or we move to the accepting state. If we read a 0, we change to q_2 and delete the 0. If we read a 1, we change to q_3 and delete the 1. Thus q_2 means that we just read a 0 and q_3 means that we just read a 1. q_2 and q_3 work very similar; thus we only describe q_2. Independent from the current symbol q_2 writes a 0, the head moves to the left and switches to q_4. Thus we are again at the blank symbol which was written by q_1 before. From q_4 we directly go to the left and change to q_1. Thus we can interpret q_1 as 'the head is at the right end of the string that we still need to move'.
This process is repeated until we have shifted the whole input and then we move to an accepting state (from q_1).

Exercise 3: Designing a Turing Machine (2+2+2+2 points)

Let $\Sigma = \{0, 1\}$. For a string $s = s_1 s_2 \dots s_n$ with $s_i \in \Sigma$ let $s^R = s_n s_{n-1} \dots s_1$ be the *reversed* string. *Palindromes* are strings s for which $s = s^R$. Then $L = \{sas^R \mid s \in \Sigma^*, a \in \Sigma \cup \{\varepsilon\}\}$ is the language of all palindromes.

(a) Give a state diagram of Turing machine recognizing L.

(b) Give the maximum number of head movements (or a close upper bound) your Turing machine makes until it halts, if started with an input string $s \in \Sigma^*$ of length $|s| = n$ on its tape.

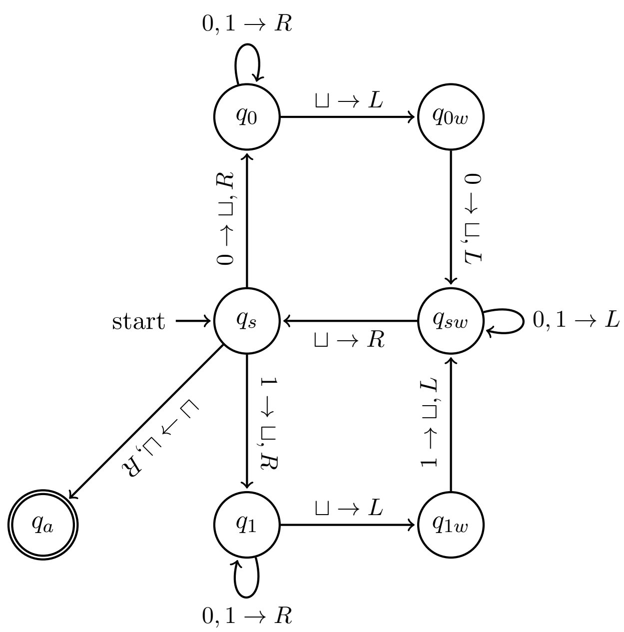
(c) Describe (informally) the behavior of a 2-tape Turing machine which recognizes L and uses significantly fewer head movements on long inputs than your 1-tape Turing machine.

(d) Give the maximum number of head movements (or a close upper bound) your Turing machine makes on any of the two tapes until it halts, if started with an input string $s \in \Sigma^*$ of length $|s| = n$ on the first tape.

Sample Solution

We describe both Turing machines by their behaviour.

1.



2. This the head of the Turing machine will move $(n+1)+n+(n-1)+\ldots+1 \leq n^2$ steps and does not need any additional space besides the input word.

3. Move the tape head of the input tape to the end, and then read backwards to the start of the input tape. As you go, write the tape symbols in order on the second tape so that you end up with the reverse of the input tape on the second tape. Reset both tape heads, and then move each to the end of the tape, at each step comparing the symbols each head is pointing to. If you find a position where the symbols are different, the input is not a palindrome and you halt-reject. If you get to the end (first blank symbol on the end) without finding a mismatch then it is a palindrome and you halt-accept.

4. The head will move three times through the input on tape 1 and twice on tape 2.